Coriolis force

As advertised in our earlier article “Centrifugal force,” in this article, we introduce another type of inertial force present in rotating frame.

If a frame is rotating in anti-clockwise direction, from the point of view of an observer in the rotating frame, he or she receives inertial force called “Coriolis force” upon moving, in the right direction with respect to the moving direction. Similarly, if a frame is rotating in clockwise direction, he or she receives “Coriolis force” in the left direction with respect to the moving direction. We will qualitatively demonstrate three of the former cases using some figures, as the later cases can be demonstrated similarly.

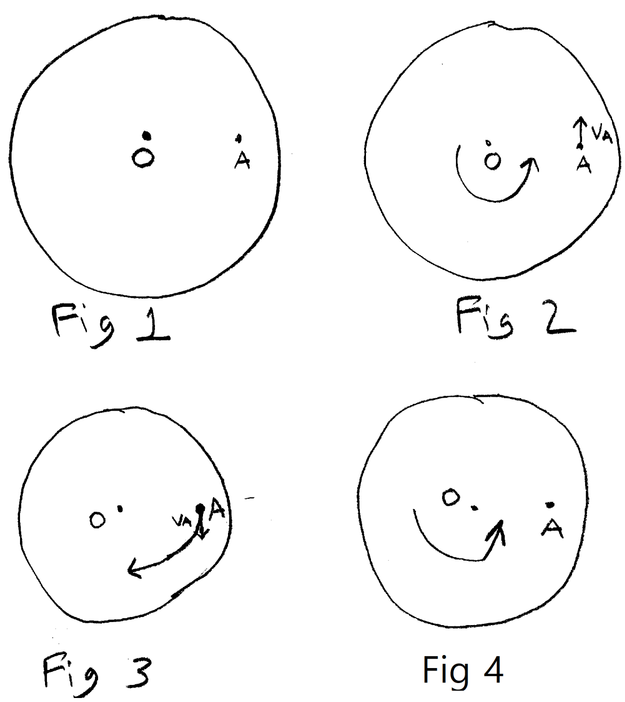
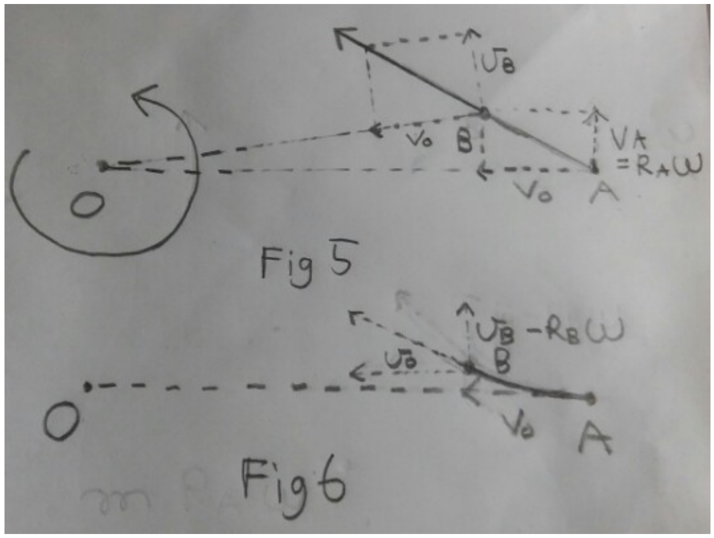


Fig.1, Fig.3, shows the point of view of the rotating frame, while Fig.2, Fig.4, shows the point of view of inertial frame. (not rotating) Notice that the disk is not seen to be rotating in Fig.1 and Fig.3 while the disk is seen to be rotating in Fig.2 and Fig.4

To begin with, an object is rest at point A in Fig.1 with respect to the rotating frame. This means that he is actually moving with speed v_A with respect to the inertial frame in Fig.2.

Now, our first example. See Fig. 4. In the inertial frame, an object is at point A and



it's not moving; the distance to the origin is constant. See Fig. 3. From the point of view of rotating frame, the object is moving at speed v_A and turn right to make the distance to the origin constant.

Second example. Fig 5 shows the point of view of the inertial frame, and Fig.6 below shows the point of view of the rotating frame. Let's say that the observer shoots a bullet toward the origin O with v_0 in the rotating frame. See Fig.6. In the inertial frame, this translates to shooting a bullet askew with velocity given by vector addition of v_A and v_0. See Fig.5. When it reaches B, the radial velocity is v_0' which is quite close to v_0 if the distance between A and B is not big. The angular velocity is v_B. We can calculate the v_B from angular momentum conservation. If the distance between O and A is R_A, and the distance between O and B is R_B, we have $mR_Av_A = mR_Bv_B$. Thus, we have $v_B > v_A$. How does this look in Fig.6? At B the radial velocity is v_0' and the angular velocity is given by $v_B - R_B\omega$. (The angular velocity at A is $v_A - R_A\omega = 0$.) (**Problem 1.** Check that $v_B - R_B\omega > 0$.) As $v_B - R_B\omega > 0$, we see that the bullet turns rightward. Before giving out the third example, let's talk about the rotation of the Earth. Everybody knows that the Sun rises in the East and sets in the West. This means that the Earth rotates from the West to the East. This means that the Earth rotates anti-clockwise in Northern hemisphere and clockwise in Southern hemisphere. Therefore, an object moving is deflected rightwards in Northern hemisphere and deflected leftwards in Southern hemisphere. For example, the wind receives such a force, and particularly, the wind blowing toward the eye of typhoon is always deflected rather than going straight toward the eye of typhoon, which implies that typhoons look like spirals.

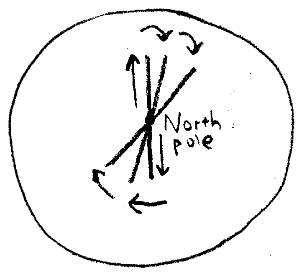


Fig 8

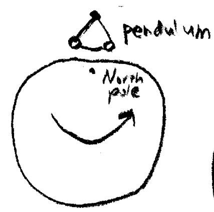


Fig 7

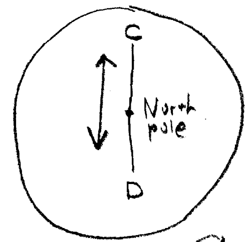


Fig 9

Our third example. See Fig.7. A pendulum is situated right above the North Pole. In Fig.9 in which the point of view of inertial frame is depicted, if the pendulum oscillates between C and D nothing changes, as much as the trajectory in Fig.4 was straight. However, from someone moving with the Earth, the oscillation trajectory of the pendulum moves clockwise as the Earth is rotating anti-clockwise. See Fig.8. In addition, you see there that this is also consistent from the Coriolis force picture. The direction of the oscillation is denoted there and it is clearly deflected rightwards. In any case, it is easy to imagine that the trajectory of the oscillation comes back to the original position after 24 hours.

Furthermore, if we situate Foucault's pendulum in northern hemisphere the trajectory of the oscillation will rotate clockwise, as it is evident from the Coriolis force picture.

Similarly, if we situate Foucault's pendulum in southern hemisphere the trajectory of the oscillation will rotate anticlockwise, as it is evident from the Coriolis force picture. Particularly, if the pendulum is situated at the South Pole, the trajectory of the oscillation will come back to the original position after 24 hours.

If Foucault's pendulum is situated in equator, it can rotate neither clockwise nor anti-clockwise. In other words, it will not rotate, or takes infinite time for the trajectory of the oscillation to come back to itself. From this point of view, Foucault's pendulum somewhere in the Northern hemisphere but not on the North Pole comes back to its original position between 24 hours and infinite time. In other words, it takes longer than 24 hours to come back to itself.

Historical comments. French mathematician and scientist Coriolis published his idea on Coriolis force in 1835. He was also the first one to derive that the kinetic energy is given by $\frac{1}{2}mv^2$. In 1851, French physicist Foucault demonstrated that the Earth rotates by showing that Foucault's pendulum rotates.

Problem 2. Which of the following is the correct answer for the hours a Foucault's pendulum at latitude θ takes to come back to its original position? (Multiple choice problem)
(a) $24\cos\theta$ (b) $24\sin\theta$ (c) $24\tan\theta$ (d) $24/\cos\theta$ (e) $24/\sin\theta$ (f) $24/\tan\theta$

We will actually approach the Coriolis force quantitatively in the next article.

Summary

- In a rotating reference frame, there are two inertial forces for a moving object: Coriolis force and centrifugal force. In this frame, if the object is not moving, there is no Coriolis force and it receives the centrifugal force only.
- The oscillating plane of Foucault's pendulum rotates due to Coriolis force. It doesn't rotate at all on the equator, and it rotates once every 24 hours on the North Pole or the South Pole.