Fermat's principle and the consistency of physics

To explain Fermat's principle, the late Nobel Laureate Richard Feynman provided the following analogy:

Suppose that a lifeguard on a beach spotted a swimmer in danger in the sea. (See Fig. 1.) The lifeguard needs to reach the swimmer as soon as possible. Which path should he take?

At first glance, you might think that the path in Fig. 2 would be the best option, since it is the shortest path. However, this choice would be incorrect. Since the lifeguard runs on the beach much faster than he swims in the sea, he needs to swim a shorter distance and run a longer one. Following this logic, you may think that the path in Fig. 3 would be the best, since he swims the shortest distance while taking this path, but this is not the case either. Think about it this way: The path the lifeguard should take depends upon the ratio between his speed on the beach and his speed in the sea. If Fig.3 were the right answer (i.e. swimming the shortest distance), it is implicitly assumed that it would still be the right answer in the limit that the swimming speed is almost the same as, but slightly lower than the running speed, which holds true as long as the swimming speed remains lower than the running speed. However, if he swims just slightly slower than he runs on the beach, the correct path would be just slightly different from the path shown in Fig. 2. Therefore, Fig. 3 cannot be the path wanted; there cannot be two paths for one limit. Therefore, we conclude that the correct answer is the one shown in Fig. 4. The lifeguard needs to swim a shorter distance than that shown in Fig. 2, but not as short a distance as the one shown in Fig. 3. Of course, the exact path is given mathematically in terms of the ratio between his swimming speed and his running speed ($\sin \theta 1/\sin \theta 2$ = running speed/swimming speed). If you know calculus, you will be able to derive this formula.

Fermat's principle, discovered in the 17th century, states that light, like an ideal lifeguard, always travels the path that takes the shortest time. Air can be compared with the beach from our analogy and water with the sea. Light changes direction at the interface between the air and the water at an angle that is given exactly by our earlier formula ($\sin \theta 1/\sin \theta 2$ = speed of light in the air/speed of light in the water). This formula is called Snell's law.

The beauty of physics is that one can obtain Snell's law in many different ways. Huygens' principle can explain Snell's law. A calculation based on the fact that the light is an electromagnetic wave can also recover Snell's law. The beauty of physics is that you can get the same result even if you have very different pictures as starting points. Furthermore, nature doesn't say that the trajectory of light obeys Snell's law because of one principle to the exclusion of other principles. All three pictures of the trajectory of light are correct, even though those who are not trained in physics are likely to mistakenly think that there can be only one explanation per phenomenon. In our later articles, we will derive Snell's law in two ways: once from Huygens' principle and once from Fermat's principle.

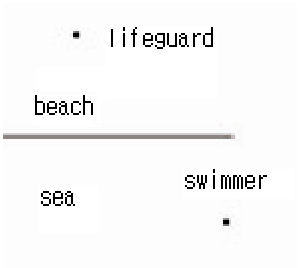

Fig. 1

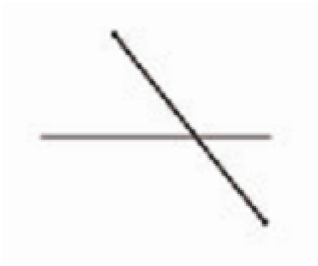

Fig. 2

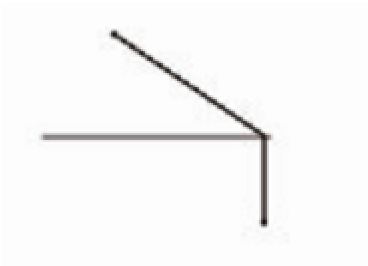

Fig. 3

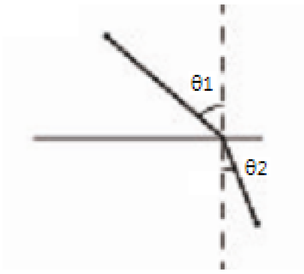

Fig. 4