Stirling's formula

Stirling's formula approximates the factorials. It is very useful.

Recall, $n! = n \times (n-1) \times \cdots 2 \times 1$. Therefore we have:

$$\ln(n!) = \ln n + \ln(n-1) + \cdots \ln 2 + \ln 1 \tag{1}$$

Then, we can approximate the sum as an integration as follows:

$$\ln(n!) = \ln n + \ln(n-1) + \cdots \ln 2 + \ln 1 \approx \int_0^n \ln x dx \tag{2}$$

Now, recall that in our earlier article "Integration by parts," we have seen:

$$\int \ln x \, dx = x \ln x - x \tag{3}$$

Plugging this in, we get[1]:

$$\ln(n!) \approx n \ln n - n \tag{4}$$

This is called "Stirling's formula." Actually, it turns out that one can do better. As an aside, we note:

$$\ln(n!) = \left(n + \frac{1}{2}\right) \ln n - n + \frac{1}{2} \ln(2\pi) + \frac{1}{12n} - \frac{1}{360n^3} + \frac{1}{1260n^5} + \cdots \tag{5}$$

Summary

- Stirling's formula approximates the factorials.
- It can be derived by calculating $\int \ln x \, dx$.

[1] Here, we used the fact that $\lim_{x \to 0} x \ln x = 0$