# Charge conjugation

In our earlier article "The symmetry of physical laws: the CPT theorem for laymen," we briefly mentioned that charge conjugation corresponds to "interchanging positive electrodes with negative electrodes and magnetic north poles with magnetic south poles." More precisely speaking, charge conjugation changes the sign of charge. For example, under charge conjugation, a particle with charge $+ke$ becomes a particle with charge $-ke$, and a particle with charge $-ke$ becomes a particle with charge $+ke$. The particle that is related by charge conjugation is called "anti-particle." For example, the anti-particle of electron is positron, which has the same mass as electron but charge $+e$ instead of $-e$ like an electron. The only property that a positron differs from an electron is its charge. We also say that electron is the anti-particle of positron, and electron and positron form a particle-antiparticle pair.

The history of discovery of positron is interesting. A couple years after Schrödinger equation was firmly established, British physicist Paul Dirac came up with Dirac equation which is a relativistic Schrödinger equation for spin 1/2 particle such as electron. (We will explain what spin 1/2 particle is in our article "Pauli matrices and spinor.") He solved the Dirac equation for himself, and found out surprisingly that the solutions of Dirac equation admit not only an electron with but also a new particle with a negative energy; while the electron has a positive energy given by $\sqrt{m^2c^4 + p^2c^2}$, this new particle has a negative energy given by $-\sqrt{m^2c^4 + p^2c^2}$. (Roughly speaking, the reason why there is a negative energy solution in addition to a positive solution is because $E^2 = m^2c^4 + p^2c^2$ has both a positive solution and a negative solution. Nevertheless, it's just hand-waving argument. It would be wrong to say that every "relativistic Schrödinger equation" has a negative energy solution.) However, this is troublesome as the negative energy $-\sqrt{m^2c^4 + p^2c^2}$ can be arbitrarily low, which would imply that the ground state energy is negative infinity. Then, this negative energy-ed particles will keep falling into states with lower and lower energy by emitting photons. This doesn't make sense. Therefore, in 1929 Dirac proposed that all the negative energy states is filled except for several states called the Dirac holes. Then, by Pauli's exclusion principle, the negative energy-ed particles cannot fall into lower energy because they are already occupied. (Of course, except for Dirac holes.) Notice also that if the energies of the states that are unfilled are given by $-E_1, -E_2, -E_3, \cdots$, such a configuration would have $E_1 + E_2 + E_3 + \cdots$ amount of energy more than the energy without any Dirac holes. So, the Dirac hole has a positive energy, and similarly a positive electric charge $+e$. The Dirac hole also has the same mass as the electron, because its energy is also given by $\sqrt{m^2c^4 + p^2c^2}$ where $m$ is the one of the electron. However, Dirac proposed that protons are anti-particles of electrons even though protons are much heavier than electrons. He guessed that they could be the anti-particles of electrons albeit with different mass by hitherto unknown mechanism.

The problem was settled when the anti-particle of electron, indeed with the same mass as electron, was discovered in 1932. It is now called positron. Dirac received Nobel prize in 1933.

In 1930s by further development of quantum field theory (i.e. relativistic version of quantum mechanics), the Dirac picture of Dirac hole was abandoned and a positron is no longer considered as a Dirac hole. We now treat positrons as real particles with positive energies.

Final comment. In our later articles on quantum field theory, we will see that particles are described "fields," a somewhat different concept than the wave function we deal in quantum mechanics. We will also see that charge conjugation corresponds to complex conjugation of fields. If you complex conjugate twice, you get yourself again. That's the reason why anti-particle of anti-particle of a particle is the original particle itself. Also, we know that if you complex conjugate a real number, you get yourself. Thus, particles that have themselves as anti-particles are described by real fields.

## Summary

- Charge conjugation makes a particle, its anti-particle.
- If you charge conjugate a particle, its sign of charge is reversed.
- If you charge conjugate twice a particle, it becomes itself. In other words, anti-particle of an anti-particle is the particle itself.