Compton scattering

An experiment performed by Compton in 1923 played a very important role in verifying the existence of light quanta called "photons." He shot light beam of certain wavelength at stationary electrons, and measured the wavelength of scattered light beam. We can easily guess that the wavelength of scattered light beam will be longer than the original wavelength, since scattered light must have less energy than the original one since some of the energy was transferred into electrons which then move, and photons with less energy have longer wavelength as the energy of photon is inversely proportional to the wavelength. (i.e. $E = hc/\lambda$) The situation is just like two-dimensional elastic collision we considered in our earlier article. The only difference is that we now have a photon and an electron to collide. Compton's experiment, which is now called "Compton scattering" verified that light, as photons, have definite energies and momentums, as his measurement agreed with the values expected in elastic collision. In this article, we will calculate the wavelength of scattered light beam in terms of the initial wavelength and the angle scattered. Unlike in our earlier article on elastic collisions, we will use relativistic formulas for energy and momentum.

Now, from our earlier article "De Broglie's matter wave" , we have:

$$E = \frac{hc}{\lambda}, \qquad p = \frac{h}{\lambda} \tag{1}$$

Given this, see Fig.1. A photon with wavelength λ travels in x-direction and scatters with a stationary electron. The scattered photon now has wavelength λ', and the scattered angle is given by θ. The electron now also moves as the arrow shows. From the conservation of

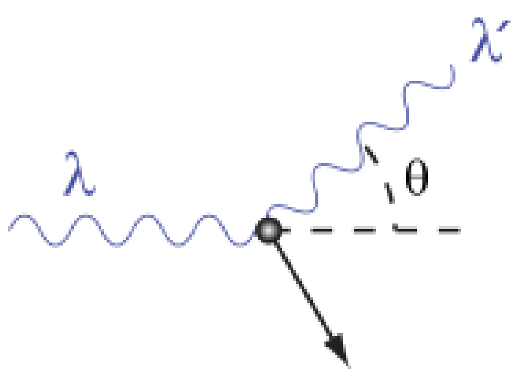

Figure 1: Comtpon scattering

energy, we have:

$$\frac{hc}{\lambda} + mc^2 = \frac{hc}{\lambda'} + E \tag{2}$$

where m is the rest mass of the electron, and E is the total energy of the electron (i.e. rest energy plus kinetic energy) after the collision. Now, let's write the conservation of momentum. The conservation of the momentum of x-component is given as follow:

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + p_x \tag{3}$$

where p_x is the momentum of the electron after the collision. The y-component yields:

$$0 = \frac{h}{\lambda'}\sin\theta + p_y \tag{4}$$

All we have to do now is solving these three equations. We have:

$$\begin{aligned}
m^2c^2 &= \frac{E^2}{c^2} - (p_x^2 + p_y^2) \\
m^2c^2 &= (\frac{h}{\lambda} + mc - \frac{h}{\lambda'})^2 - (\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta)^2 - \frac{h^2}{\lambda'^2}\sin^2\theta \\
m^2c^2 &= \frac{h^2}{\lambda^2} + m^2c^2 + \frac{h^2}{\lambda'^2} + 2mc(\frac{h}{\lambda} - \frac{h}{\lambda'}) - \frac{2h^2}{\lambda\lambda'} - \frac{h^2}{\lambda^2} + \frac{2h^2}{\lambda\lambda'}\cos\theta - \frac{h^2}{\lambda'^2}\cos^2\theta - \frac{h^2}{\lambda'^2}\sin^2\theta \\
0 &= 2mc(\frac{h}{\lambda} - \frac{h}{\lambda'}) - \frac{2h^2}{\lambda\lambda'} + \frac{2h^2}{\lambda\lambda'}\cos\theta = \frac{2mch(\lambda' - \lambda)}{\lambda\lambda'} - \frac{2h^2}{\lambda\lambda'}(1 - \cos\theta)
\end{aligned}$$

Therefore, we conclude:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) \tag{5}$$

This is the final result. We indeed see that the wavelength increases. (i.e. $\lambda' > \lambda$)

(The figure is from http://en.wikipedia.org/wiki/File:Compton-scattering.svg)

Summary

- Compton scattering shows that a photon is indeed a particle with a definite momentum and energy.