Cylindrical coordinate system

In our earlier articles, we have studied the polar coordinate. We have also seen that the polar coordinate was useful in obtaining planet's orbit around the Sun. In this article, we will introduce the cylindrical coordinate system, a coordinate system in 3d and hybrid of the polar coordinate system and the Cartesian system.

The cylindrical coordinate (r, θ, z) is defined as follows:

$$x = r\cos\theta \tag{1}$$
$$y = r\sin\theta \tag{2}$$
$$z = z \tag{3}$$

See Fig. 1. r is the distance to the z-axis. Notice that x, y coordinates are given by the ones of polar coordinate, and z coordinate is the one for the Cartesian coordinate.

Now, some application of cylindrical coordinate system. The 2-dimensional surface given by

$$r = R \tag{4}$$

is an infinite cylinder, as the collection of points equidistant to the z-axis is a cylinder. Another way of seeing this is this. See Fig. 2. If you cut the cylinder along the plane parallel to $x - y$ plane (i.e. z =constant) then the intersection is a circle with radius R. So, this must be a cylinder.

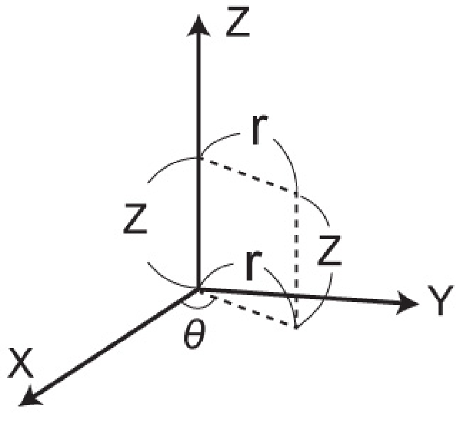

Figure 1: the cylindrical coordinate system

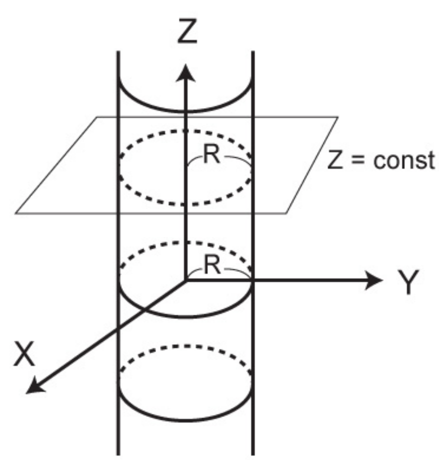

Figure 2: $r = R$, a cylinder

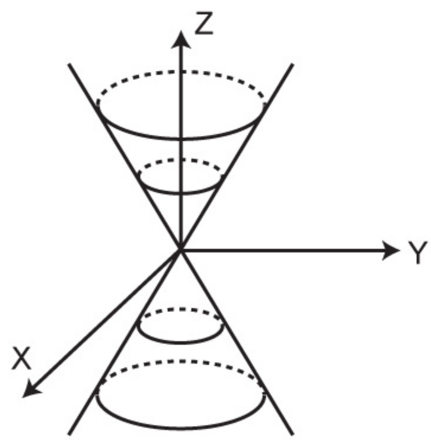

Figure 3: $r = z$, a cone

Another example. The 2-dimensional surface given by

$$r = z \tag{5}$$

is a cone. If you cut the cone along the plane parallel to $x - y$ plane (i.e. $z = z_0$) then the intersection is a circle with radius z_0; the farther from the $x - y$ plane the bigger the radius of the circle.

Summary

- Cylindrical coordinate is a 3-dimensional coordinate that is a hybrid of polar coordinate and Cartesian coordinate.
- $x = r\cos\theta, \qquad y = r\sin\theta, \qquad z = z$