Electric potential revisiting

In our earlier article "Kinetic energy and Potential energy in three dimensions, Line Integrals and Gradient," we introduced potential energy and the concept of conservative force. We will apply these concepts to electric field in this article.

In our earlier article "Electric field," we defined electric field as:

$$\vec{E} = \frac{\vec{F}}{q} \tag{1}$$

Given this, let's integrate the both-hand side along a curve. As we know that the integration of force is potential energy U, we obtain:

$$\int \vec{E} \cdot d\vec{s} = \frac{\int \vec{F} \cdot d\vec{s}}{q} = \frac{U}{q} \tag{2}$$

This suggests that we can define electric potential V as follows:

$$V = -\int \vec{E} \cdot d\vec{s} \tag{3}$$

In other words, what force is to electric field is what potential energy is to electric potential. By this analogy, convince yourself following:

$$\vec{E} = -\nabla V \tag{4}$$

As much as a ball moves from a high place (i.e. a place with high potential energy) to a low place, a positive charge moves from a place with high electric potential to a place with low electric potential, and a negative charge moves from a place with low electric potential to a place with high electric potential. This suggests that electric current from a place with high electric potential to a lower one.

Also, I want to remark that electric potential is well-defined in absence of changing magnetic field since it is a conservative force in such cases. We will consider the case that the magnetic field changes in our later article "electric potential and vector potential."

Final remark. Volt is the unit for electric potential. You may be familiar with this unit, if you use electric gadgets and sockets.

Problem 1. Show that the electric potential at a point r from an object with charge Q is given as follows: (This is called "Coulomb potential." Hint[1])

$$V = k\frac{Q}{r} \tag{5}$$

[1]Use (3). Our earlier articles "Revisiting Gausss law and the derivation of Coulombs law" and "Kinetic energy and Potential energy in polar coordinate" can be helpful.

In particular, show that the potential energy of a system with two charges q_1 and q_2 with distance r apart is given by:

$$U = k\frac{q_1 q_2}{r} \tag{6}$$

Problem 2. Let's say charge Q is uniformly distributed on a hollow sphere with radius R. Then, what is the electric potential on the surface of sphere? What is the electric potential inside the sphere at the location $R/2$ far from the center? (Hint[2])

Problem 3. Consider Fig.2 in our earlier article "Revisiting Gauss's law and the derivation of Coulomb's law." Calculate the electric potential at the point P. Find also the electric field there by using (4). This must agree with the answer you have obtained there.

Problem 4. Consider Fig.3 in our earlier article "Revisiting Gauss's law and the derivation of Coulomb's law." Calculate the electric potential at the point P. Find also the electric field there by using (4). This must agree with the answer you have obtained there.

Summary

- $\vec{E} = -\nabla V$. In other words,
$$V = -\int \vec{E} \cdot d\vec{s}$$
- $V = k\frac{Q}{r}$

[2] First, show that outside the sphere, the electric field is the same as the case in which all the charge is at the center. Second, think about what the electric field inside the sphere is. Then, obtain the answer by using (3) and the convention that one sets electric potential at infinitely far point to zero.