Exponents

Consider the following quantity:

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \tag{1}$$

It is too long: is there a simpler way to express this? There is. We write:

$$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \tag{2}$$

where 6 here denotes the fact that 3 is multiplied 6 times. Here, we call 3 the "base" and 6 the "exponent." We pronounce 3^6 as "3 to the power of 6."

Problem 1. Express the followings using exponent notation.

$$5 \times 5 \times 5 = ? \qquad (-2) \times (-2) \times (-2) \times (-2) = ?$$

In mathematics, we often see cases in which the exponent is 2 or 3, so we have special ways to pronounce such expressions. For example, instead of pronouncing 5^2 as "5 to the power of 2", we often pronounce it as "5 squared." Similarly, we often pronounce 5^3 as "5 cubed."

Problem 2. What is 5 squared? What is 2 cubed?

Problem 3. Evaluate the following.

$$3^4 = ?, \qquad (-2)^3 = ?, \qquad \left(\frac{1}{3}\right)^2 = ?$$

Problem 4. In this problem, we will revisit the rule that we have learned in the last article. Notice that $(-1)^2 = 1$, $(-1)^3 = -1$, $(-1)^4 = 1$, $(-1)^5 = -1$ and so on. From this pattern, evaluate $(-1)^{2016}$ and $(-1)^{4349}$.

Another way of obtaining this expression is following. If n is an integer, $(-1)^{2n} = \big((-1)^2\big)^n = 1^n = 1$.

Similarly, $(-1)^{2n+1} = (-1)^{2n}(-1)^1 = 1 \cdot (-1) = -1$.

Can an exponent be a negative number? For example, what is 2^{-3}? At first glance, this seems to be a senseless question, as we cannot certainly multiply a number negative times. However, it turns out that we can assign a value to a negative exponent consistently. We will talk about it in "Exponents, revisited."

Can an exponent be a fraction? For example, what is $9^{1/2}$? At first glance, this seems to be a senseless question, as we cannot certainly multiply a number half times. However, again, it turns out that we can assign a value. We will talk about it in "Root, cube root, and nth root, revisited."