Graphs of quadratic polynomials

In this article, we will consider the graph of the form $y = ax^2 + bx + c$.

Before doing so, let's first consider simpler cases where $y = ax^2$. Remember that we have considered this kind of graph when $a > 0$ in Problem 4 of "The Cartesian coordinate system and graph." The lesson there was that y increases as x goes to negative infinity or positive infinity. This holds true for a graph $y = ax^2 + bx + c$ for $a > 0$ as well. (You can see this if you complete the square.) Also, the y-minimum point is called the "vertex." In the case of $y = ax^2$, we have seen that the vertex was at $(0, 0)$.

Now let's consider $y = ax^2$ when $a < 0$.

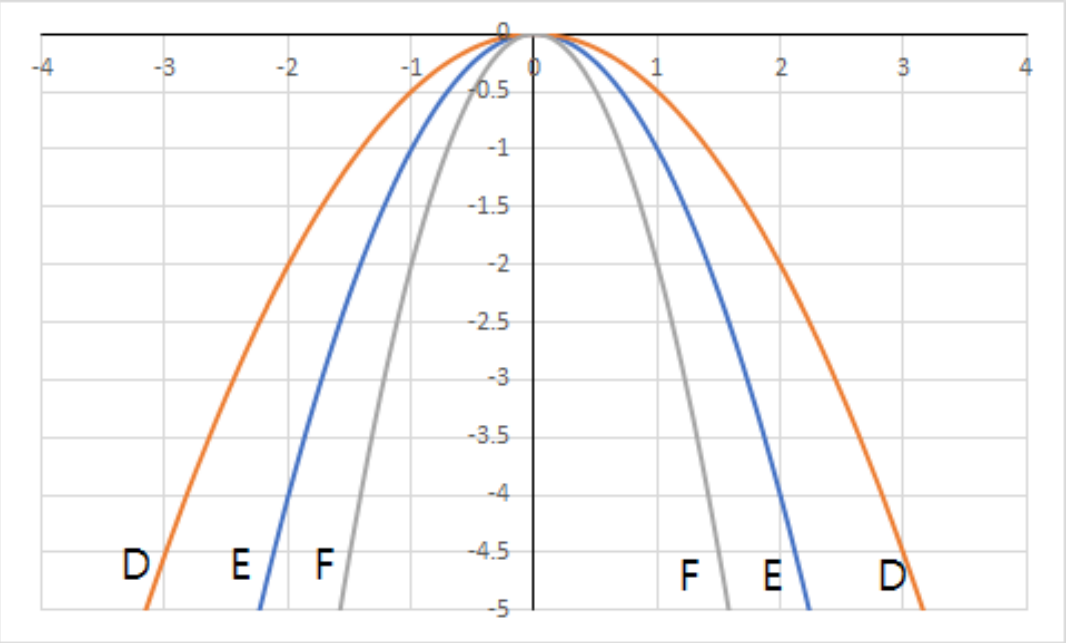

Figure 1: $y = -x^2, -2x^2, -\dfrac{x^2}{2}$

Problem 1. In Fig.1 we have three graphs: D, E, F. They are $y = -x^2, y = -2x^2, y = -\dfrac{x^2}{2}$. Which one is which? Notice that y can be never positive for all three graphs as something squared multiplied by a negative number is always negative or 0. The maximum for y, 0 is achieved when x

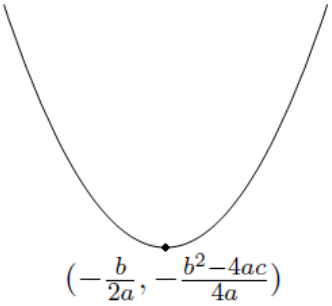

Figure 2: $y = ax^2 + bx + c, a > 0$

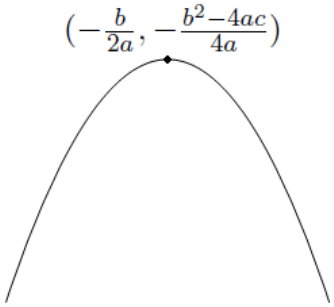

Figure 3: $y = ax^2 + bx + c, a < 0$

is 0. For a graph with $y = ax^2 + bx + c$ for $a < 0$, the y-maximum point is called the "vertex." In our case, the vertex is at $(0, 0)$

Now, let's consider a general case. We consider $a > 0$ first. Completing the square, we get

$$y = ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a} \tag{1}$$

Notice that the minimum value the term $a(x + \frac{b}{2a})^2$ can have is 0, and the minimum is achieved when $x + \frac{b}{2a} = 0$. Therefore, the minimum of (1) is $-\dfrac{b^2 - 4ac}{4a}$ and achieved when $x = -\dfrac{b}{2a}$. Therefore, the vertex is at $(-\dfrac{b}{2a}, -\dfrac{b^2 - 4ac}{4a})$. See Fig.2.

Problem 2. Check that the vertex of $y = ax^2 + bx + c$ for $a < 0$ is also at $(-\dfrac{b}{2a}, -\dfrac{b^2 - 4ac}{4a})$. See Fig.3.

Summary

- For a graph $y = ax^2 + bx + c$ for $a > 0$, y increases as x goes to negative infinity or positive infinity. The y-minimum point is called the "vertex."
- For a graph $y = ax^2 + bx + c$ for $a < 0$, y decreases as x goes to negative infinity or positive infinity. The y-maximum point is called the "vertex."
- You can obtain the coordinate of vertex by completing the square.