Inscribed triangle in a circle

Given a triangle, there always exists unique circle in which the triangle is inscribed. See Fig. 1. (In other words, for every triangle there is only one, neither zero nor two, circle that passes its three vertices.) In this article, we will prove why.

We will prove this by construction i.e., by finding such a circle. See Fig. 2. Let's find such a circle for $\triangle ABC$. D is the midpoint of $\overline{AB}$, and E is the midpoint of $\overline{AC}$. Given this, from the point D, let's draw a line, which is perpendicular to $\overline{AB}$. Similarly, from the point E, let's draw a line, which is perpendicular to $\overline{AC}$. Then, they meet at G. These two lines are denoted by dotted lines. Then, it is easy to see that $\triangle AGB$ and $\triangle AGC$ are isosceles triangles. Thus

$$\overline{BG} = \overline{AG}, \qquad \overline{AG} = \overline{GC} \tag{1}$$

which implies

$$\overline{AG} = \overline{BG} = \overline{GC} \tag{2}$$

Thus, we see that we can draw a circle centered at G and passes all the three points A, B, C. In other words, (2) implies that these three points are located same distance from the center of circle.

Let me make a comment. From the second equality of (2), we see that $\triangle BGC$ is also an isosceles triangle. Thus, if we draw an altitude from G to $\overline{BC}$ it bisects $\overline{BC}$, and meet at its middle point denoted in the figure by F. In other words, if we draw three perpendicular lines from the three midpoints, they meet at one point.

Summary

- Given any triangle, there always exists unique circle in which it is inscribed.

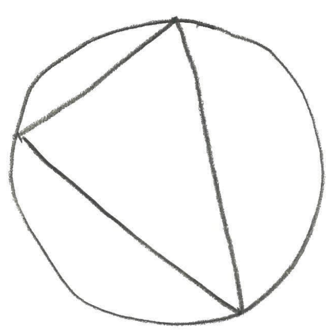

Figure 1: an inscribed triangle in a circle

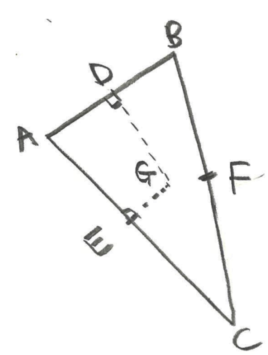

Figure 2: finding the center of the circle