Addition and subtraction of negative numbers

Suppose you have 10 dollars, and buy a 7 dollar earphone. How much dollars do you have now? It is $10 - 7 = 3$ dollars. Suppose further that somebody gives you 5 dollars now. How much money do you have? It's $3 + 5 = 8$ dollars.

Let's consider now a different situation. Suppose you have 4 dollars, and buy a 7 dollar earphone. How much dollars do you have now? Of course, you do not have enough money to buy the earphone. But, let's say that the earphone seller still agrees to hand over the earphone, on the condition that you pay all the money you have, namely 4 dollars, and pay him 3 dollars later, so that he can eventually get the full 7 dollars for the earphone later. Then, you will owe him 3 dollars. So, how much dollars do you have now? You have 3 dollars in debt. There is a convenient way to express this. $4 - 7 = -3$. We pronounce -3 as "negative three." Here, we call $-$ in -3, "the negative sign." Suppose further that somebody gives you 5 dollars now. How much money do you have? With 5 dollars, you can now pay 3 dollars to the earphone seller, then you will have 2 dollars left. We can express this as $-3 + 5 = 2$.

Let's briefly summarize what we have just learned. We have

$$4 - 7 = -3 \tag{1}$$

which you can deduce from $7 - 4 = 3$. We have

$$-3 + 5 = 2 \tag{2}$$

which you can deduce from $5-2 = 3$. Notice further that we always have $a+b = b+a$, where a and b are numbers. For example, $4+2 = 2+4$. Therefore, we need to have $-3+5 = 5+(-3)$. Thus, we have

$$5 + (-3) = 2 \tag{3}$$

because $5 - 3 = 2$. In other words, adding -3 is the same thing as subtracting 3. More generally, adding $-n$ is the same thing as subtracting n.

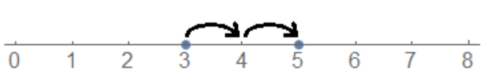

Figure 1: $3 + 2 = 5$

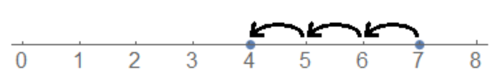

Figure 2: $7 - 3 = 4$

Actually, it is possible to visualize all these calculations. To begin with, let's consider simple, familiar examples. $3 + 2 = 5$ can be represented as Fig. 1 and $7 - 3 = 4$ can be represented as Fig. 2. We see that adding 2 corresponds to moving 2 spaces to the right, and subtractig 3 corresponds to moving 3 spaces to the left.

Now, let's calculate $4-7$. But how can we subtract 7 from 4? You cannot move 7 spaces to the left from 4, as there is no grid on the left side of 0. The trick is introducing new grids on the left side of 0. The new grids represent numbers with negative signs. See Fig. 3. As we go to the left, we have $0, -1, -2, -3$ and so on. Given this, $4-7$ is possible. If you have to move 7 spaces to the left from 4, you can move 4 spaces to the left first, which lands you at 0, and 3 further spaces to the left, which lands you at -3. Thus, we have $4-7=-3$.

See Fig. 4 for $-3+5$. You have to move 5 spaces to the right from -3. If you move 3 spaces to the right, you will land at 0. If you move 2 spaces further, to complete the 5 space movement, you will land at 2. $-3+5=2$.

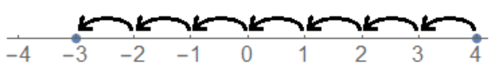

Figure 3: $4-7=-3$

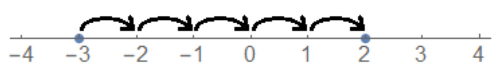

Figure 4: $-3+5=2$

Now, we turn to subtracting negative numbes. To this end, remember that $a+b=c$ implies $a=c-b$, where a, b and c are numbers. For example, $4+2=6$ implies $4=6-2$. Therefore, $5+(-3)=2$ implies $2-(-3)=5$. Notice here that we can calculate $2-(-3)$ by calculating $2+3=5$. In other words,

$$2-(-3)=5 \tag{4}$$

because $2+3=5$. In other words, subtracting -3 is the same thing as adding 3. More generally, subtracting $-n$ is the same thing as adding n.

Consider also, you have 4 dollars in debt. In other words, you have -4 dollars. Suppose somebody gives you 3 dollars. How much money do you have now? You can use this 3 dollars to pay part of your debt. Then, you will have now 1 dollar left in debt. In other words,

$$-4+3=-1 \tag{5}$$

which you can deduce from $4-3=1$. From the rule $a+b=b+a$, we need to have $-4+3=3+(-4)$. Thus,

$$3+(-4)=-1 \tag{6}$$

Another way of seeing this is using the fact that adding -4 is the same thing as subtracting 4. Thus,

$$3+(-4)=3-4=-1 \tag{7}$$

Now, let's consider another situation. Suppose you have 1 dollar in debt. But, you buy a 3 dollar ice cream promising to pay later. Then you have 4 dollar in debt. This means

$$-1-3=-4 \tag{8}$$

See Fig. 5 for a graphical representation of this calculation. If you move 3 spaces to the left from -1, you land at -4. Another way of seeing (8) is from (5). $a+b=c$ implies $a=c-b$. Thus, $-4+3=-1$ implies $-4=-1-3$.

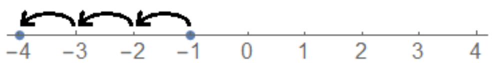

Figure 5: $-1-3=-4$

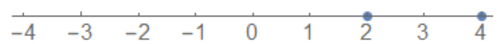

Figure 6: $4>2$

Given this, suppose you need to calculate $-1+(-3)$. Recalling that adding -3 is the same thing as subtracting 3, we have

$$-1+(-3)=-1-3=-4 \tag{9}$$

where we used (8) in the last step.

Let me ask you a question. Michael has 4 dollars, and Jane has 2 dollars. Who has more money? Michael, because 4 is bigger than 2. In other words,

$$4>2 \tag{10}$$

See Fig. 6. 4 is on the right side of 2.

Now, let's say Irene has 0 dollar, and Jonathan has -2 dollars (i.e. 2 dollars in debt). Who has more money? Irene. Jonathan still needs to pay 2 dollars, while Irene has nothing to pay. This suggests

$$0>-2 \tag{11}$$

See Fig. 7. 0 is on the right side of -2.

Suppose now Carson has -3 dollars (i.e. 3 dollars in debt), and Benjamin has -1 dollars (i.e. 1 dollars in debt). Who has more money? The one who has less in debts. Therefore, we obtain

$$-1>-3 \tag{12}$$

See Fig. 8. -1 is on the right side of -3. In conclusion, if $a>b$, on the grid, a is on the right side of b.

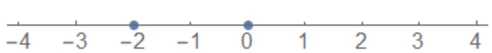

Figure 7: $0>-2$

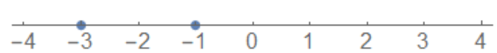

Figure 8: $-1>-3$

Any number that is bigger than 0 is called a "positive number," and any number that is smaller than 0 is called a "negative number." Any number that is not a negative number is called "non-negative number." 0 and positive numbers are non-negative numbers. On the grids, positive numbers are on the right side of 0, and negative numbers are on the left side of 0. Before learning the concept of negative numbers, you have only regarded positive numbers and zero as numbers. Sometimes, it is convenient to put + sign (called "positive sign") in front of positive number, even though it is more or less redundant. For example, instead of writing $4 + 5 = 9$, one can write

$$+4 + (+5) = 9 \tag{13}$$

It is important that you get used to the kind of calculations presented in this article. To this end, you must know these three following rules by heart.

- $a + (-b) = a - b$ ex) $5 + (-3) = 5 - 3 = 2.$ (14)
- $a - (-b) = a + b$ ex) $5 - (-3) = 5 + 3 = 8.$ (15)
- $-a + (-b) = -(a + b)$ ex) $-3 + (-2) = -(3 + 2) = -5$ (16)

Problem 1.

$$1 - 2 =?, \qquad 2 - 4 =?, \qquad 4 - 7 =?, \qquad 5 - 6 =?$$

Problem 2.

$$3 - 2 =?, \qquad 3 - 6 =?, \qquad 3.4 - 4 =?, \qquad 3.49 - 3.5 =?$$

Problem 3.

$$3 + (-4) =?, \qquad 4 + (-7) =?, \qquad 5 + (-6) =?, \qquad 10 + (-8) =?$$

Problem 4.

$$-1 + (-3) =?, \qquad -2 + (-4) =?, \qquad -3 + (-7) =?, \qquad -6 + (-6) =?$$

Problem 5.

$$(-8) + 2 =?, \qquad (-8) + 11 =?, \qquad (-3) + 10 =?, \qquad (-3) + 6 =?$$

Problem 6.

$$3 - (-1) =?, \qquad 1 - (-2) =?, \qquad 4 - (-2) =?, \qquad 5 - (-3) =?$$

Problem 7.

$$-4 - 3 =?, \qquad -3 - 4 =?, \qquad -4 - 7 =?, \qquad -2 - 1 =?$$

Problem 8.

$$-3 - (-1) =?, \qquad -1 - (-2) =?, \qquad -4 - (-2) =?, \qquad -5 - (-3) =?$$

Problem 9.

$$-1-(-1)=?, \qquad -1+(-1)=?, \qquad -3-(-2)=?, \qquad -3-3=?$$

Problem 10.

$$\frac{1}{2}+\left(-\frac{1}{3}\right)=?, \qquad \frac{1}{3}-\frac{1}{2}=?, \qquad -\frac{1}{3}-\frac{1}{2}=?$$

Problem 11.

$$-\frac{1}{2}-\left(-\frac{1}{4}\right)=?, \qquad -\frac{1}{3}+(-\frac{1}{2})=?$$