Light ray reflecting on a mirror

In our earlier article "Fermat's principle and the consistency of physics," we have explained that we can apply Fermat's principle to find the light ray in refraction. (Refraction is a phenomenon in which light ray or wave changes their direction when entering different media.) In this article, we will apply Fermat's principle to find what the light ray looks like when light ray reflects off at a mirror.

When you shoot a light ray into a mirror the angle of reflection is equal to the angle of incidence. See Fig. 1. The angle of incidence is denoted by θ_i and the angle of reflection by θ_r. They satisfy $\theta_i = \theta_r$. Our object of this article is proving this relation step by step. See Fig. 2. The light ray intends to travel from A to B by bouncing at a point on the surface of the mirror. There are many trajectories the light *could* have followed. For example, it could bounce at C, C_1 or C_2. Then, the length of trajectory is given by $\overline{AC} + \overline{CB}$ or $\overline{AC_1} + \overline{C_1B}$ or $\overline{AC_2} + \overline{C_2B}$ depending on the bouncing point. Fermat's principle says that the actual path of light ray is $A \to C \to B$, if $\overline{AC} + \overline{CB}$ is the shortest path among all the trajectories with all the other possible bouncing points on the surface of the mirror such as C_1 or C_2. So, we will first find such C, then show that such C satisfies $\theta_i = \theta_r$. To this end, see Fig. 3. B' is the mirror image of B. In other words, $\overline{BD} = \overline{B'D}$ and $\overline{BB'}$ is perpendicular to the surface of the mirror.

Problem 1. By using the congruency of triangle (covered in our article "Congruence of triangle"), show that $\overline{C_1B'} = \overline{C_1B}$, $\overline{CB} = \overline{CB'}$, $\overline{C_2B} = \overline{C_2B'}$.

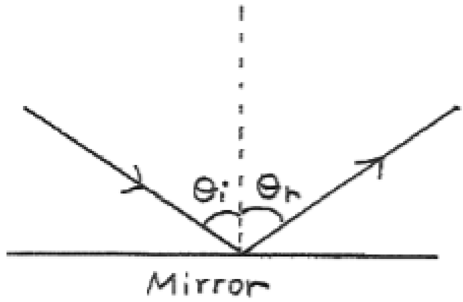

Figure 1: $\theta_i = \theta_r$

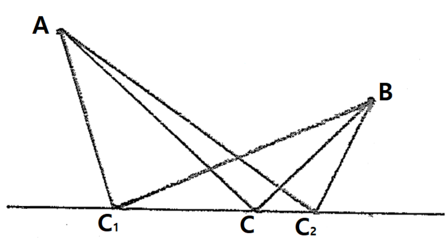

Figure 2: the shortest path is the actual path

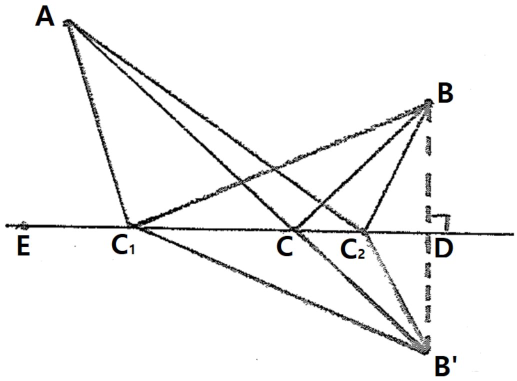

Figure 3: Finding the shortest path through B'

Thus, we have

$$\overline{AC_1} + \overline{C_1B} = \overline{AC_1} + \overline{C_1B'} \tag{1}$$

and similarly for C_2 and C.

Then, as far as the distance of the trajectory is concerned, we can just find the shortest path among the paths that go from A to B' passing the surface of the mirror instead of the shortest path that goes from A to B by reflection. Then, it is easy to see that C must lie on the line $\overline{AB'}$, because it is the shortest path.

Problem 2. Using the triangle inequality, explain why

$$\overline{AC} + \overline{CB'} < \overline{AC_1} + \overline{C_1B'} \tag{2}$$

$$\overline{AC} + \overline{CB'} < \overline{AC_2} + \overline{C_2B'} \tag{3}$$

are satisfied.

Problem 3. Explain why $\angle ACE = \angle BCD$.

As $\theta_i = 90° - \angle ACE$ and $\theta_r = 90° - \angle BCD$, we conclude $\theta_i = \theta_r$.