Scientific notation

What is the mass of the Earth?
It is 5,972,000,000,000,000,000,000,000 kg.

What is the mass of the electron?
It is 0.000 000 000 000 000 000 000 000 000 000 910 938 36 kg

You may now notice that it is very cumbersome to count number of zeros in the above numbers to figure out how big the mass of the Earth is exactly and how small the mass of the electron is exactly. Is there a better way to express these numbers? Yes. It is called "scientific notation" and uses the "exponents" we just learned in the last article. The mass of the Earth is 5.972×10^{24} kg and the mass of the electron is $9.1093836 \times 10^{-31}$ kg. This way, you do not need to count the zeros at all to figure out how big and how small these numbers are. The factors 10^{24} and 10^{-31} already have the relevant information. So, a scientific notation is in following form:

$$m \times 10^n \tag{1}$$

where n is an integer, and m is equal to or more than 1 and less than 10. For example, 52,500 in scientific notation is 5.25×10^4 and 0.0490 is 4.90×10^{-2}.

Problem 1. Express the following numbers using scientific notation.

$$0.003455 =? \qquad 79.8 =? \qquad 1,345 =? \tag{2}$$

Of course, for such moderate numbers (i.e., neither enormously big nor enormously small), the usefulness of the scientific notation may not be apparent as our first examples (the Earth's mass and the electron's mass). However, there is still a big advantage which I am going to explain now.

Most measurements involve some errors. For example, if you measure the size of your textbook using a ruler of which the minimum gradation is 0.1 cm, you cannot measure the size of your textbook better than 0.1 cm. For example, if you obtained that the size of your textbook is 25.3 cm using such a ruler, it only means that the real size lies between 25.25 cm and 25.35 cm. Scientists usually denote this size as (25.3 ± 0.05) cm.

Now, you may know that 25.3 and 25.300 are the same number. Or, are they? In science and engineering, they may mean slightly different things. If you obtained that the size of your textbook is 25.3 cm using the ruler we just mentioned it is of no use to say that the size of your textbook is

25.300 cm because you never know whether it is 25.334 cm or 25.297 cm. In such a case the two zeros at the end of 25.300 cm is not only useless but also misleading. So, you better write the size of your textbook as "25.3 cm" instead of "25.300 cm." However, suppose you use a better measurement apparatus to determine the size of your textbook. This time, the minimum gradation is 0.001 cm and suppose you actually obtained 25.300 cm. Then, the actual size is (25.300 ± 0.0005) cm. This time, it's worth writing it as "25.300 cm" instead of "25.3 cm." Thus, 25.3 cm and 25.300 cm may mean different things in science and engineering unlike in math.

Suppose now a ship weighs 82,700,000 kg. Again, this weight has some errors as no scale can measure an object without any error. However, we cannot be sure of the error of the weight of the ship written in this way. Does this mean that the ship weighs between 82,650,000 kg and 82,750,000 kg? or between 82,699,500 kg and 82,700,500 kg? or between 82,699,999.5 kg and 82,700,000.5 kg? You may think that it might be the last one, but that's not always how we use the English language when we say a number. For example, when you say that the US population according to 2015 census was 309 millions (309,000,000) you do not actually mean that the US population was exactly 309 millions not deviating even one person, but with some errors. Even though the census said that the US population was 308,745,538, you often round it to 309 millions and say that the US population was 309 millions.

Anyhow, if you express the weight of the ship using scientific notation the case is clear. If the weight of the ship is 8.27×10^7 kg, it means that it is between 8.265×10^7 kg and 8.275×10^7 kg. If the weight of the ship is 8.2700×10^7 kg it means that it is between 8.26995×10^7 kg and 8.27005×10^7 kg. Thus, you see the advantage of scientific notation.

Actually, what I have said so far about errors is roughly correct but not exactly correct. So far, the value of the errors (such as 0.05 cm, 0.0005 cm, 0.005×10^7 kg, 0.00005×10^7 kg) all began with the numeral 5. However, in reality, the measurement error is not always like that. For examples, the mass of the Earth is $(5.9722 \pm 0.0006) \times 10^{24}$ kg. Thus, outside of science, it is not that wrong to say that the mass of the Earth is 5.972×10^{24} kg as it is between 5.9716×10^{24} kg and 5.9728×10^{24} kg.[1] So, I just wanted to point out that, when we say that the mass of the Earth is 5.972×10^{24} kg, it does not automatically mean that it is between 5.9715×10^{24} kg and 5.9725×10^{24} kg, but just roughly lies around that range.

Another common notation is the following: $5.9722(6) \times 10^{24}$ kg means $(5.9722 \pm 0.0006) \times 10^{24}$ kg. Sometimes, the error can be expressed more accurately by two digits instead of one digit. For example, the mass of the electron is $9.10938356(11) \times 10^{-31}$ kg. Notice that the error (11) has two

[1] Strictly speaking, there is a small chance that the actual value still lies off this range, but I would not confuse the readers too much by explaining these details at this point.

digits. As $56-11=45$ and $56+11=67$, the mass of the electron is between $9.10938345 \times 10^{-31}$ kg and $9.10938367 \times 10^{-31}$ kg. Of course, if you do not bother to be accurate about the error you can express this using one digit error instead of two digit error as $9.1093835(1) \times 10^{-31}$ kg, which means it is between $9.1093834 \times 10^{-31}$ kg and $9.1093836 \times 10^{-31}$ kg.