System of linear equations, part I: two unknowns

There are a total of 7 dogs and pigeons. If the total number of legs on these animals is 20, how many dogs and how many pigeons do we have?

Notice that there are two unknowns in this problem: the number of dogs and the number of pigeons. So, this type of equation is new. So far we have only considered equations with only one unknown, which we labeled x. Nevertheless, we can solve this new type of equation easily. This is the goal of this article.

Let's write down the equations. If the number of dogs is x and the number of pigeons is y we have:

$$\begin{aligned} x + y &= 7 \quad (1) \\ 4x + 2y &= 20 \quad (2) \end{aligned}$$

The trick is to get rid of one of the variables in the equation. To do so, let's express y in terms of x. From the first equation, we have:

$$y = 7 - x \quad (3)$$

Then, we can plug this into the second equation. We get:

$$4x + 2(7 - x) = 20 \quad (4)$$

What we have done is convert two equations with two unknowns to one equation with one unknown. In any case, if you solve this equation, you will get $x = 3$. Plug this back into (3), we have $y = 4$. So, 3 dogs and 4 pigeons is the answer. We can also check that we got the right answer. $3 + 4 = 7$, $4 \times 3 + 2 \times 4 = 20$

Of course, we could solve this problem alternatively: you could express x in terms of y instead of expressing y interms of x. Then, you have $x = 7 - y$. Plugging this in, we get:

$$4(7 - y) + 2y = 20 \quad (5)$$

If you solve this, you will get $y = 4$. As $x = 7 - y$ we have $x = 3$. Of course, it is guaranteed that you will get the same answer.

There is still another method to solve this problem which is sometimes easier. Multiply both sides of (1) by 4. Then, we get:

$$\begin{aligned} 4x + 4y &= 28 \quad (6) \\ 4x + 2y &= 20 \quad (7) \end{aligned}$$

where in the second line we reproduced (2) for convenience. If you subtract the second line from the first line, we get:

$$2y = 8 \quad (8)$$

Therefore, we get $y = 4$. Plugging this into (1), we get:

$$x + 4 = 7 \quad (9)$$

Therefore, we get $x = 3$. Now, notice that we multiplied (1) by 4 so that the coefficient in front of x, namely, 4, matches the $4x$ in (2). Thus, we could eliminate the variable x upon subtraction.

Of course, this is not the only way to solve this problem. We could eliminate y. For example, if we divide (2) by 2, we get:

$$\begin{aligned} 2x + y &= 10 \quad (10) \\ x + y &= 7 \quad (11) \end{aligned}$$

where in the second line we reproduced (1) for convenience. If you subtract the second line from the first line, we get $x = 3$. plugging this back to (11), we get:

$$3 + y = 7 \quad (12)$$

So, we get $y = 4$. Notice here that we divided (2) by 2, so that $2y$ there becomes y which then matches the term y in (11). This allows us to eliminate y.

In later articles, we will see how to solve a system of equations with three or more variables. We will also see when one has infinite solutions or when one has no solution at all, given a system of equations.

Problem 1. Solve the following equations.

$$\begin{aligned} 2(2x + y) &= 3x + 4 \\ x + 1 &= 3y \end{aligned}$$

Problem 2. Solve the following equations.

$$\begin{aligned} 2x + 2 &= 3y \\ 3x - 2y &= 2 \end{aligned}$$

Problem 3. Solve the following equations. (Hint[1])

$$\begin{aligned} \frac{3a+b}{a+2b} &= 1 \\ a+b &= 3 \end{aligned}$$

Problem 4. Solve the following equations.

$$10F - (10+15)W = 0$$
$$F - 30 - W = 0$$

Problem 5. Catherine's grandfather is twice as old as her. If he was thrice as old as her 25 years ago, what are Catherine's age and her grandfather's age now?

Problem 6. A proton is made out of two "up" quarks and one "down" quark. A neutron is made out of one "up" quark and two "down" quarks. The electric charge of a proton is $+e$ while the electric charge of a neutron is 0. Using the fact that an electric charge of a particle is the sum of the electric charge of its constituents, find the electric charges of an "up" quark and a "down" quark.

Problem 7. Express v_{1f} and v_{2f} in terms of other variables. We will encounter these equations in our later article on collisions.

$$\begin{aligned} v_{1f} - v_{2f} &= v_{2i} - v_{1i} \\ m_1 v_{1i} + m_2 v_{2i} &= m_1 v_{1f} + m_2 v_{2f} \end{aligned}$$

Problem 8. Let's say any x is a solution to the following equation. (i.e. any x satisfies the equation.) Then, what are the values of n and m?

$$(n+m)x + n = 4x + m \qquad (13)$$

Summary

- When you solve a system of linear equations that have two unknowns, you need to get rid of one of the unknowns first. Then, you can solve the resulting linear equation of one unknown. Once you obtain the unknown so, you can plug it in to one of the linear equations to get the other unknown.

[1] The first equation implies $3a + b = a + 2b$.