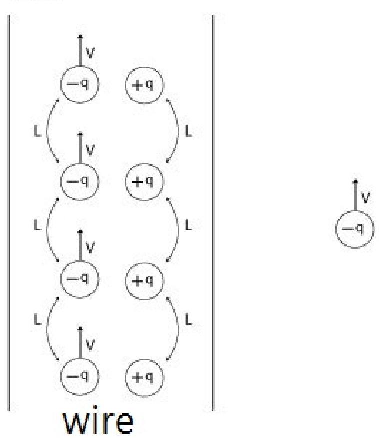
Origin of the magnetic force from the perspective of special relativity.

Why is there such a thing as magnetic force? Couldn't God have chosen the laws of nature to have only electric forces without magnetic forces?

In this article, I will argue that a world with electric forces but without magnetic forces would be inconsistent with Einstein's theory of special relativity. This claim is really a tautology, since the theory of special relativity is based on the fact that the speed of light – or equivalently, of the electromagnetic wave – is always constant in a vacuum; that is, since the theory of special relativity is based on the electromagnetism. However, because it makes for an interesting exercise, let's momentarily forget the fact that light is an electromagnetic wave, and "derive" the fact that in a world where electric forces exist, and where the speed of light is always constant (or equivalently, where the theory of special relativity is correct), magnetic forces are required.

Consider the following figures that show charged particles moving through a wire and another charged particle outside the wire.

Figure 1.



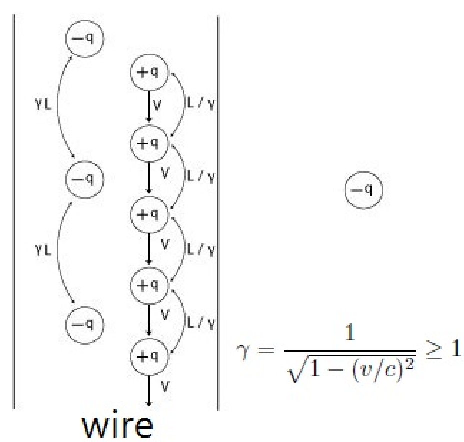
Suppose first that as in figure 1, there is a wire with electric current passing downward, and a nearby object with charge $-q$ moving upward with speed v. (Here, for simplicity, I assume that the electric current is carried by particles with charge $-q$, so that the direction that charges move is opposite to the direction of electric current. Moreover, I assume that these charged particles move with speed v, so that their velocity is the same as that of the object moving upward. These configurations are for convenience and may not accurately represent the real world, but luckily, the reasoning I will use can be easily generalized.)

Notice that the wire in figure 1 carries not only negatively charged particles which are moving, but also positively charged particles which are at rest. I included the positively charged particles in the wire to make it electrically neutral. Remember that the electric wires we see in our daily lives are always electrically neutral, even when electric currents pass through them.

Given this situation, what would be the force exerted upon the charged object that's moving upward beside the wire? If you have studied some high school physics, you know that in our world, an electric current produces magnetic fields around it, and a charged object moving in certain directions is affected by those fields. But if there were no such thing as magnetic force, the object moving upward wouldn't receive any force.

If this were the case, however, another observer must agree on the lack of force as well. Let's check whether this would be the case. Consider an observer who is also moving upward, with the same velocity as the charged object. He will observe our configuration not as in figure 1 but as in figure 2; from his perspective, the charged object and the negatively-charged particles in the wire won't move, while the positively-charged particles in the wire will move downward.

Figure 2.



Note that there would be Lorentz-Fitzgerald contraction going on in this situation; the distance between adjacent positively-charged particles which the observer in figure 2 observes would be smaller than that between positively-charged particles observed by an observer in figure 1, since the positively-charged particles are moving from the perspective of the observer in figure 2 but are at rest from the perspective of the observer in figure 1. Conversely, the distance between adjacent negatively-charged particles which an observer in figure 1 observes will be smaller than that between negatively-charged particles observed by the observer in figure 2, since negatively charged particles are moving from the perspective of the observer in figure 1 and are at rest from the perspective of the observer in figure 2. The distances between the particles are denoted in the figures. In figure 1, the distances between the negatively charged particles or the positively charged particles are both L, whereas in figure 2, they are γL and L/γ, respectively.

All this leads to the surprising conclusion that the electric wire observed by the observer in figure 2 is no longer electrically neutral; it is positively charged, because the negatively-charged particles are sparser than the positively-charged particles. Therefore, the observer in figure 2 would observe an electrical force, and would observe that the electric wire attracts the object with charge $-q$.

If this attraction between the electric wire and the charged object really exists, it would also be observed by the observer in figure 1, even though the two observers may not agree on the magnitude of the attracting force. However, the electric wire is electrically neutral from the perspective of the observer in figure 1, so the nature of the force acted upon the object by the electric wire cannot be electrical. This force must be something other than electric force. We call this magnetic force.

Thus, we have showed that a world with electric force but without magnetic force is inconsistent with special relativity.[1] Of course, the fact that an electric current induces a magnetic force on charges moving in certain direction was well known well before the advent of special relativity. Nevertheless, it's always interesting to see things from different angles.

[1] In fact, the transformation laws of electric fields and magnetic fields between two different observers were derived in Einsteins famous paper on special relativity published in 1905. However, unlike this article, his derivation was mathematical rather than physical.