

The mathematical beauty of physics: simplicity, consistency, and unity

A British physicist Paul Dirac says of mathematical beauty as following:

"It is quite clear that beauty does depend on one's culture and upbringing for certain kinds of beauty, pictures, literature, poetry and so on...But mathematical beauty is of a rather different kind. I should say perhaps it is of a completely different kind and transcends these personal factors. It is the same in all countries and at all periods of time."

The law of physics possesses the mathematical beauty Dirac talked about. So, what is the mathematical beauty of physics? I propose that the law of physics is mathematically beautiful because it has three qualities: simplicity, consistency, and unity.

First, simplicity. It is just amazing how after a long night of calculations every ugly term cancels out and a complicated string of symbols boils down to an equation a single line long. This simplicity appears like a miracle from God. Maybe God truly is behind it. I think this must be what Albert Einstein meant in saying, "When the solution is simple, God is answering."

In 1861, Maxwell published "On Physical Lines of Forces" in which he established Maxwell's equations, which were collections of laws that govern everything of the electromagnetic fields. His equations were twelve complicated equations. In 1884, Heaviside reduced them to four simple equations. In 1908, Minkowski reduced it to two. When I learned in freshman math class homework how to reduce the four Maxwell's equation to the simple two equations, I was mesmerized. " $dF=0$, $d^*F=j$ " This is the simplicity of physical laws. Notice that Maxwell didn't know these simpler and more beautiful versions of his equations, when he first created them. The fact that this simplification has been possible demonstrates that there is "something," a mathematical reality.

Second, The consistency of physics—the ability to arrive at the same conclusion using different methods—seems to me to be another one of God's miracles. For example, Heisenberg's quantum mechanics and Schrödinger's quantum mechanics are equivalent, even though they may seem very different.¹ The actual calculations involved in each are very different, but amazingly, they give the same results.

Similarly, Feynman's quantum electrodynamics looks quite different from those of Tomonaga and Schwinger, but each set of calculations gives the same answer². Yet another example: Joseph

¹ Schrödinger proved this equivalency after months of work.

² Freeman Dyson proved this equivalency.

Polchinski writes in the preface of "String Theory" that "...the critical dimension of the bosonic string is calculated in seven different ways in the text and exercises" in his book.

As Richard Feynman, Nobel Laureate in Physics, says, "Every theoretical physicist who is any good knows six or seven different theoretical representations for exactly the same physics."³ I find this degree of consistency to be particularly beautiful.

Finally, let me explain about a third beautiful quality of physics: unity. Albert Einstein once said:

"Creating a new theory is not like destroying an old barn and erecting a skyscraper in its place. It is rather like climbing a mountain, gaining new and wider views, discovering unexpected connections between our starting points and its rich environment. But the point from which we started out still exists and can be seen, although it appears smaller and forms a tiny part of our broad view gained by the mastery of the obstacles on our adventurous way up."

Every new theory in physics must be able to explain new phenomena *in addition to* describing old phenomena that an old theory has already explained adequately. This requirement constrains the development of new theories in a very strong way.

Physics is not a patchwork endeavor in which you can look at the results of new experiments independent from all others, coming up with separate formulas and theories to explain each of them. Albert Einstein once said, "A theory can be proved by experiment, but no path leads from experiment to the birth of a theory."⁴ You first have to make a theory consistent with the old and verified theories, calculate what kind of experimental results the new theory predicts, and then compare the predicted results with the experimentally attained ones. This is what I mean by the beauty of "unity."

Carlo Rovelli and Francesca Vidotto wrote in their book:

"Contradiction between empirically successful theories is not a curse: it is a terrific opportunity. Several of the major jumps ahead in physics have been the result of efforts to resolve precisely such contradictions. Newton discovered universal gravitation by combining Galileo's parabolas with Kepler's ellipses. Einstein discovered special relativity to solve the "irreconcilable" contradiction between mechanics and electrodynamics. Ten years later, he discovered that spacetime is curved in an effort to reconcile Newtonian gravitation with special relativity. Notice

³ Feynman, Richard. (2001). *The Character of Physical Law* (24th ed., p. 168). Cambridge, Massachusetts: The M.I.T. Press.

⁴ The Sunday Times (18 Jul 1976).

that these and other major steps in science have been achieved without virtually any *new* empirical data. Copernicus for instance constructed the heliocentric model and was able to compute the distances of the planets from the Sun using only the data in the book of Ptolemy.”⁵

Indeed, it's amazing how pure logic can lead to new theories. Regarding general relativity and Yang-Mills theory, which are fundamental to contemporary particle physics, Fundamental Physics Prize Laureate Nima Arkani-hamed said:

“So if you just hand a bunch of theorists the laws of relativity and quantum mechanics they are confident, that if you lock them up in a room, you don't let them look at what the world looks like outside, and just ask what could the world look like, this is what they will come up with.”

Although general relativity was discovered in a different fashion, Yang-Mills theory actually was developed without any experimental input.

Johann Wolfgang Goethe said, “Beauty is a manifestation of secret natural laws, which otherwise would have been hidden from us forever.” Certainly, if there were no beauty in physics laws, the Yang-Mills theory would have never been discovered, especially since it could not be deduced from experiments.

To conclude this article, let me quote Freeman Dyson:

“On being asked what he meant by the beauty of a mathematical theory of physics, Dirac replied that if the questioner was a mathematician then he did not need to be told, but were he not a mathematician then nothing would be able to convince him of it.”

Actually, it is hard to see the beauty of physics at a low level such as high school level or college underclassman level. Murray Gell-Mann, a Nobel laureate in physics recalled the following story.⁶ When he applied to Yale at the age of 14, he was thinking of putting down archaeology or linguistics as an intended major in the application form. However, his father advised against it, telling him that he would starve, and suggested him to study engineering. Murray Gell-Mann replied that he would rather starve. So, his father suggested him to study physics instead. He replied that he had taken a physics course in high school which had been not only the dullest course he had ever taken but also the only one in which he had ever done badly. His father replied “It will be very different when you take advanced courses in physics. You'll learn general

⁵ Rovelli, Carlo, and Francesca Vidotto. (2015). *Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity and Spianfoam Theory* (p. 5). Cambridge University Press.

⁶ John Brockman, ed., *Curious Minds: How a Child Becomes a Scientist* (New York: Pantheon Books, ©2004)

relativity and quantum mechanics. And those are very, very beautiful." To satisfy his father, he put down physics as the intended major thinking that he could change his major later. However, once he entered Yale, he was too lazy to switch major and got hooked on quantum mechanics and relativity.

So, don't think that physics is dull because you studied physics at high school and did badly on it. It is still possible that you get hooked on quantum mechanics and relativity later, and become the next Gell-Mann.

The more you study physics, the more its simplicity, consistency, and unity will convince you of its truth. You will be sure that aliens, had they had a civilization, would have deduced Newton's laws and Einstein's theory of relativity just as we did. I am not sure the same could be said about the facts of biology or psychology. That's why I love physics the most out of all the sciences.