

Addition and subtraction of negative numbers

Suppose you have 10 dollars, and buy a 7 dollar earphone. How much dollars do you have now? It is $10 - 7 = 3$ dollars. Suppose further that somebody gives you 15 dollars now. How much money do you have? It's $3 + 15 = 18$ dollars.

Let's consider now a different situation. Suppose you have 5 dollars, and buy a 7 dollar earphone. How much dollars do you have now? Of course, you do not have enough money to buy the earphone. But, let's say that the earphone seller still agrees to hand over the earphone, on the condition that you pay all the money you have, namely 5 dollars, and pay him 2 dollars later, so that he can eventually get the full 7 dollars for the earphone later. Then, you will owe him 2 dollars. So, how much dollars do you have now? You have 2 dollars in debt. There is a convenient way to express this. $5 - 7 = -2$. We pronounce -2 as "negative two." Here, we call $-$ in -2 , "the negative sign." Suppose further that somebody gives you 15 dollars now. How much money do you have? With 15 dollars, you can now pay 2 dollars to the earphone seller, then you will have 13 dollars left. We can express this as $-2 + 15 = 13$.

Let's briefly summarize what we have learned so far. We have

$$5 - 7 = -2 \tag{1}$$

which you can deduce from $7 - 5 = 2$. We have

$$-2 + 15 = 13 \tag{2}$$

which you can deduce from $15 - 2 = 13$. Notice further that we always have $a + b = b + a$, where a and b are numbers. For example, $4 + 2 = 2 + 4$. Therefore, we need to have $-2 + 15 = 15 + (-2)$. Thus, we have

$$15 + (-2) = 13 \tag{3}$$

because $15 - 2 = 13$.

At this point, remember that $a + b = c$ implies $a = c - b$, where a , b and c are numbers. For example, $4 + 2 = 6$ implies $4 = 6 - 2$. Therefore, $15 + (-2) = 13$ implies $13 - (-2) = 15$. Notice here that we can calculate $13 - (-2)$ by calculating $13 + 2 = 15$. In other words,

$$13 - (-2) = 15 \tag{4}$$

because $13 + 2 = 15$.

Consider also, you have 4 dollars in debt. In other words, you have -4 dollars. Suppose somebody gives you 3 dollars. How much money do you have now? You can use this 3 dollars to pay part of your debt. Then, you will have now $4 - 3 = 1$ dollar in debt. In other words,

$$-4 + 3 = -1 \tag{5}$$

which you can deduce from $4 - 3 = 1$. From the rule $a + b = b + a$, we need to have $-4 + 3 = 3 + (-4)$. Thus,

$$3 + (-4) = -1 \tag{6}$$

Another way of seeing this is following. Remember $15 + (-2) = 15 - 2 = 13$. Similarly, we have

$$3 + (-4) = 3 - 4 = -1 \tag{7}$$

Now, let's consider another situation. Suppose you have 1 dollar in debt. But, you buy a 3 dollar ice cream promising to pay later. Then you have 4 dollar in debt. This means

$$-1 - 3 = -4 \tag{8}$$

Another way of seeing this is from (5). $a + b = c$ implies $a = c - b$. Thus, $-4 + 3 = -1$ implies $-4 = -1 - 3$.

Now, let's look at (3). We saw that $15 - 2 = 13$ implies $15 + (-2) = 13$. In other words, $15 + (-2) = 15 - 2$. More generally, $a + (-b) = a - b$. Thus, if we need to calculate $-1 + (-3)$, we can calculate it using this property.

$$-1 + (-3) = -1 - 3 = -4 \tag{9}$$

where we used (8).

Let me ask you a question. Michael has 5 dollars, and Jane has 3 dollars. Who has more money? Michael, because 5 is bigger than 3. In other words,

$$5 > 3 \tag{10}$$

Now, let's say Irene has 0 dollar, and Jonathan has -2 dollars (i.e. 2 dollars in debt). Who has more money? Irene. Jonathan still needs to pay 2 dollars, while Irene has nothing to pay. This suggests

$$0 > -2 \tag{11}$$

Suppose now Carson has -6 dollars (i.e. 6 dollars in debt), and Benjamin has -2 dollars (i.e. 2 dollars in debt). Who has more money? The one who has less in debts. Therefore, we obtain

$$-6 < -2 \tag{12}$$

Any number that is bigger than 0 is called a "positive number," and any number that is smaller than 0 is called a "negative number." Before learning the concept of negative numbers, you have only regarded positive numbers and zero as numbers. Sometimes, it is

convenient to put + sign (called “positive sign”) in front of positive number, even though it is more or less redundant. For example, instead of writing $4 + 5 = 9$, one can write

$$+4 + (+5) = 9 \quad (13)$$

It is important that you get used to the kind of calculations presented in this article. To this end, you must know these three following rules by heart.

$$\bullet a + (-b) = a - b \quad \text{ex) } 5 + (-3) = 5 - 3 = 2. \quad (14)$$

$$\bullet a - (-b) = a + b \quad \text{ex) } 5 - (-3) = 5 + 3 = 8. \quad (15)$$

$$\bullet -a + (-b) = -(a + b) \quad \text{ex) } -3 + (-2) = -(3 + 2) = -5 \quad (16)$$

Problem 1.

$$1 - 2 = ?, \quad 2 - 4 = ?, \quad 4 - 7 = ?, \quad 5 - 6 = ?$$

Problem 2.

$$3 - 2 = ?, \quad 3 - 6 = ?, \quad 3.4 - 4 = ?, \quad 3.49 - 3.5 = ?$$

Problem 3.

$$3 + (-4) = ?, \quad 4 + (-7) = ?, \quad 5 + (-6) = ?, \quad 10 + (-8) = ?$$

Problem 4.

$$-1 + (-3) = ?, \quad -2 + (-4) = ?, \quad -3 + (-7) = ?, \quad -6 + (-6) = ?$$

Problem 5.

$$(-8) + 2 = ?, \quad (-8) + 11 = ?, \quad (-3) + 10 = ?, \quad (-3) + 6 = ?$$

Problem 6.

$$3 - (-1) = ?, \quad 1 - (-2) = ?, \quad 4 - (-2) = ?, \quad 5 - (-3) = ?$$

Problem 7.

$$-4 - 3 = ?, \quad -3 - 4 = ?, \quad -4 - 7 = ?, \quad -2 - 1 = ?$$

Problem 8.

$$-3 - (-1) = ?, \quad -1 - (-2) = ?, \quad -4 - (-2) = ?, \quad -5 - (-3) = ?$$

Problem 9.

$$-1 - (-1) = ?, \quad -1 + (-1) = ?, \quad -3 - (-2) = ?, \quad -3 - 3 = ?$$

Problem 10.

$$\frac{1}{2} + \left(-\frac{1}{3}\right) = ?, \quad \frac{1}{3} - \frac{1}{2} = ?, \quad -\frac{1}{3} - \frac{1}{2} = ?$$

Problem 11.

$$-\frac{1}{2} - \left(-\frac{1}{4}\right) = ?, \quad -\frac{1}{3} + \left(-\frac{1}{2}\right) = ?$$