Ampere's law as corrected in Maxwell's equations

We know that divergence of curl of any vector is zero. Therefore, if the vector concerned is magnetic field, using Ampere's law, we have:

$$0 = \nabla \cdot (\nabla \times B) = \mu_0 \nabla \cdot J \tag{1}$$

However, from continuity equation, we know that the right-hand side is not zero, but equal to $-\mu_0 \frac{\partial \rho}{\partial t}$. In the 19th century, this was noticed by Maxwell, one of the greatest physicist in all time, and he changed Ampere's law as follows:

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{2}$$

Now, let's check the divergence of this one:

$$\begin{aligned} \nabla \cdot (\nabla \times \vec{B}) &= \mu_0 (\nabla \cdot \vec{J} + \epsilon \frac{\partial \nabla \cdot \vec{E}}{\partial t}) \\ &= \mu_0 (\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t}) = 0 \end{aligned} \tag{3}$$

where from the first line to the second line we used Gauss's law. Therefore, this correction insures that the charge is conserved. It means that this correction is necessary. Given this, what is the implication of this correction? Consider, Faraday's law, which we re-write here for convenience:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{4}$$

This equation means that electric field is induced if magnetic field changes. By comparing (2) with this one, we can easily see that (2) means that magnetic field is induced if electric field changes.

Final remark. Along with two Gauss's laws for electric field and for magnetic field, (2) and (4) are called "Maxwell's equations." Maxwell was the first one who expressed these equations in terms of derivatives. However, he didn't use the modern notation such as curl and divergence. Instead, he expressed it components by components. For example, instead of (4), he wrote:

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t} \tag{5}$$

and two other similar equations for B_x and B_y. Maxwell found out these laws using diagrams, such as Fig.1. It was English physicist Heaviside, who expressed all these using curl and divergence.

(The figure is from `http://en.wikipedia.org/wiki/File:Molecular_Vortex_Model.svg`)

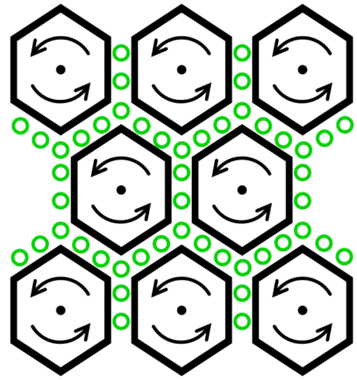

Figure 1: Maxwell's diagram

Problem 1. How should be Ampere's law expressed using integration form (i.e. $\int \vec{B} \cdot d\vec{A} = \mu_0 i$) corrected? Use the notation $\Phi_E = \int \vec{E} \cdot d\vec{A}$ for the electric flux.

Summary

- $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$