The CMB today

In our third article on history of astronomy, we explained that the Cosmic Microwave Background Radiation (CMB) originated during recombination. Anyhow, during the recombination, the wavelength (or, equivalently, frequency) distribution of photons (i.e., light) obeyed Planck's blackbody radiation spectrum (apart from very tiny anisotropies). Since then, the universe has expanded, which means that the wavelength distribution of the universe has changed because of the red-shift. So, you may naively think that the CMB now doesn't follow Planck's blackbody radiation spectrum anymore. However, it does. In this article, we will show why.

From our earlier article on blackbody radiation, we know that, in our Universe, at the moment of recombination, there were following number of photons with frequency between f_r and $f_r + df_r$:

$$dN = \frac{8\pi f_r^2 df_r}{c^3} \frac{V_r}{\exp(hf_r/kT_r) - 1} \tag{1}$$

where "r" denotes recombination and V_r is the volume of our Universe then.

We know that the number of CMB photons are, to a very good approximation, conserved. The CMB photons that we observe last scattered during the recombination, and they did not scatter since; they just went straight without being destroyed or generated.

Also, remember that the wavelength of photons are red-shifted due to the expansion of universe by the factor of scale factor. If we denote the frequency of CMB photon now by f_0, we have

$$f_0 = \frac{a_r}{a_0} f_r \tag{2}$$

as the frequency is inversely proportional to the wavelength. As the number of CMB photons are conserved, by plugging (2) into (1), we obtain that the number of photons between frequency f_0 and $f_0 + df_0$ now is given by

$$dN = \frac{8\pi f_0^2 df_0}{c^3} \left(\frac{a_0}{a_r}\right)^3 \frac{V_r}{\exp(hf_r/kT_r) - 1} \tag{3}$$

However, notice that the volume of universe now is given by

$$V_0 = \left(\frac{a_0}{a_r}\right)^3 V_r \tag{4}$$

Thus, (3) can be re-written as

$$dN = \frac{8\pi f_0^2 df_0}{c^3} \frac{V_0}{\exp(hf_0/kT_0) - 1} \tag{5}$$

if we define T_0 by the relation

$$\frac{hf_0}{kT_0} = \frac{hf_r}{kT_r} \tag{6}$$

In other words,

$$T_0 = \frac{a_r}{a_0} T_r \tag{7}$$

Now, notice that (5) is precisely the form of Planck black body radiation spectrum. The only difference from (1) is that the indice r is replaced by the indice 0. In other words, the CMB still follows the black body radiation spectrum even though our Universe expands. From (7), we also see that the CMB temperature of our Universe drops as it expands.

Summary

- Even though our Universe expands, the distribution of the wavelength of CMB photons still follows Planck's blackbody radiation spectrum.
- The CMB temperature drops as our Universe expands. It is inversely proportional to the scale factor.