Discrete area spectrum and the black hole entropy I: Loop quantum gravity and the Bekenstein-Hawking entropy

In 1973, Bekenstein suggested that a black hole should have entropy proportional to the area of its event horizon. In 1976, Hawking calculated the black hole entropy and confirmed that the black hole entropy is indeed proportional to its area. He obtained the formula

$$S = \frac{kA}{4l_p^2} \tag{1}$$

where S is the entropy, k is the Boltzmann constant, l_p is the Planck length (which is approximately 1.62×10^{-35} cm), and A is the area of the black hole's event horizon. This equation is called the Bekenstein-Hawking entropy formula.[1] Hawking obtained it by considering $S = \Delta Q/T$ which we introduced in our earlier article "What is entropy? From a macroscopic point of view." On the other hand, we know that we can approach entropy from a microscopic point of view, by counting the number of microstates. That's not something that Hawking did, but Rovelli did from the context of loop quantum gravity.

Problem 1. In an earlier article, we have seen $S = k \ln W$. This shows that entropy has the same dimension as k. Check that the Bekenstein-Hawking entropy formula (1) also has the same dimension as k as it should, if it is a formula for an entropy.

I will soon explain what the number of microstates means in the case of a black hole in the context of loop quantum gravity, but before doing so, let me explain what loop quantum gravity says about the allowed value for area, also known as "area spectrum."[2]

Loop quantum gravity predicts that the value of the area is only allowed to be a sum of specific unit areas. Notice that there are multiple unit areas, not a single one. Rather than trying to accurately describe what the actual values of the allowed area are, let me give you some examples that help to illustrate what the discrete area spectrum means. Let's say that the following are unit areas:

$$0.3\text{cm}^2, 0.4\text{cm}^2, 0.6\text{cm}^2, 0.7\text{cm}^2, 1\text{cm}^2, 1.3\text{cm}^2, 1.4\text{cm}^2, 1.6\text{cm}^2, 2\text{cm}^2 \cdots \tag{2}$$

Then, the area could not have values such as 0.35cm^2 or 0.694cm^2 , since these cannot be made as a sum of unit areas. In other words, a statement such as "the surface area of this ball is 0.694cm^2" wouldn't make any sense, because it cannot be made from the unit areas.

[1] See our earlier two articles on entropy, if you don't know what entropy is.

[2] You are encouraged to read the article Atoms of Space and Time by Lee Smolin, one of the fathers of loop quantum gravity. The article is available at http://www.phys.lsu.edu/faculty/pullin/sciam.pdf. Nevertheless, in this article, I will not assume that you have already read this.

However, 0.9cm^2 is allowed, because the surface can have two unit-area components, whose areas are 0.3cm^2 and 0.6cm^2.

In other words, the area spectrum is discrete since there are only certain values of area allowed. It would have been continuous, but not discrete, if all values were allowed for area.

Of course the actual values of unit areas are very small. They are about 10^{50} times smaller than the surface area of an atom. That's the reason why we don't notice the discreteness of the spectrum of area.

Now, as advertised, let me explain what the number of microstate means in the context of black hole entropy. In 1996, Rovelli proposed that the number of microstates is the number of possible ways that the area of the black hole can be expressed as the sum of the values of unit areas.

For example, let's say that the area of a black hole is 1cm^2. Then the number of microstates is 8, as there are eight ways of producing this area using sums of unit areas:

$$\begin{aligned}
1 &= 1 \\
1 &= 0.7+0.3 \\
1 &= 0.6+0.4 \\
1 &= 0.4+0.6 \\
1 &= 0.4+0.3+0.3 \\
1 &= 0.3+0.7 \\
1 &= 0.3+0.4+0.3 \\
1 &= 0.3+0.3+0.4
\end{aligned}$$

Then the entropy of this black hole is given by:

$$S = k \ln 8 \tag{3}$$

from Boltzmann's entropy formula $S = k \ln W$.

Given this, what does it mean to predict the black hole entropy formula using loop quantum gravity? By comparing the Bekenstein-Hawking entropy formula and Boltzmann's entropy formula, one can easily conclude the following relation:

$$\ln W = \frac{A}{4l_p^2} \tag{4}$$

In other words,[3]

$$W = e^{\frac{A}{4l_p^2}} \tag{5}$$

Therefore, if we prove that the number of possible ways to express the black hole area A as sums of unit areas in the discrete area spectrum is given by the above formula for W, then we are done with predicting the black hole entropy by using loop quantum gravity.

[3] e is a number which is very important in mathematics, and is given by approximately 2.718.

We need to prove this, because physics must be self-consistent, and the black hole entropy value derived from Hawking's way needs to be the same as the one derived by loop quantum gravity.

However, loop quantum gravity physicists have not completely determined the values of the unit areas; they have obtained the relative ratio between the unit areas, but not their absolute values. Therefore, there is a free parameter in the values of the unit area. They make this parameter fixed by assuming that the values of the unit area correctly predict the black hole entropy. Since there are no other known ways to determine this free parameter, a consistent check is lacking.

Nevertheless, in 2009 I proposed a set of new unit areas that is free of such a free parameter; I obtained not only the relative ratio between the unit areas, but also their absolute values. I plugged these values into a numerical formula which should give the value "1" if the above equation W is satisfied. I obtained approximately 0.997 for this formula. Therefore, it seems that my new unit areas are almost correct. Then, I conjectured that this difference 0.003 is due to extra dimensions which seem to modify the unit areas.

On the other hand, string theorists Strominger and Vafa, now both at Harvard, successfully showed in 1996 that string theory correctly predicts the Bekenstein-Hawking entropy for a specific kind of black hole. Their derivation had nothing to do with the unit areas, and they counted the number of different kind of microstates than the ones in loop quantum gravity. Of course, their method is equally valid.

Therefore, the Bekenstein-Hawking entropy formula still needs to be understood in terms of loop quantum gravity. I hope that my derivation of the Bekenstein-Hawking entropy using my new unit area will be accepted by the loop quantum gravity community. It has the potential to be confirmed once the Large Hadron Collider, an accelerator that began operation in November 2009, produces miniature black holes.

Summary

- The Bekenstein-Hawking entropy says that the entropy of black hole is given by

$$S = \frac{kA}{4l_p^2}$$

- Loop quantum gravity predicts that the value of the area is only allowed to be a sum of specific unit areas.

- According to loop quantum gravity the number of microstates for a black hole is the number of possible ways that the area of the black hole can be expressed as the sum of the values of unit areas.