Discrete area spectrum and the black hole entropy II: Domagala-Lewandowski-Meissner formula

In the last article, I explained the relation between the discrete area spectrum and the black hole entropy. In this article, I will derive a formula which must be satisfied if the black hole entropy is given by the Bekenstein-Hawking entropy formula once the discrete area spectrum is given.

As explained in the last article, there are only certain allowed values for the area of a black hole, because there is a discrete spectrum for the area. In mathematical terms, if there are three fundamental unit areas A_1, A_2, and A_3, we can express all the allowed values of the area as

$$A = c_1 A_1 + c_2 A_2 + c_3 A_3 \tag{1}$$

where c_1, c_2, c_3 are some non-negative integers. For example, consider a hypothetical situation in which A_1 is 0.3 cm^2, A_2, 0.4 cm^2, and A_3, 0.5cm^2, and that c_1 is 3, c_2 is 4, and c_3 is 1. Then, $3 \times 0.3\text{cm}^2 + 4 \times 0.4\text{cm}^2 + 1 \times 0.5\text{cm}^2 = 3\text{cm}^2$ is an allowed value for the area. This area has $8 = 3 + 4 + 1$ compartments, the area of each of which is a fundamental unit area. Given this, how many possible ways can the area 3 be expressed as a sum using the discrete area spectrum? Lets find a way to systemically obtain the number of such possible sums. We will use $W(A)$ to refer to the number of possible ways to express the given area A as a sum from the discrete area spectrum.

Now, observe that if a unit area is part of a sum of unit areas totaling A, then A can be expressed as that unit area plus the rest of the area. In our example, the allowed area value 3cm^2 can include a compartment with any of the three unit areas, and thus can be written in each of the following ways:

$$3 = 0.3 + 2.7$$
$$3 = 0.4 + 2.6$$
$$3 = 0.5 + 2.5$$

Therefore, we can write:

$$W(3) = W(3 - 0.3) + W(3 - 0.4) + W(3 - 0.5) \tag{2}$$

since the number of possible ways to express 3 as the sums of the discrete area spectrum is equal to the sums of the numbers of ways possible ways to express 2.7, 2.6 and 2.5 each as the sums of the discrete area spectrum.

Now, recall that the number of possible ways to express the area of a black hole A as a sum using the discrete area spectrum is given by the following formula:

$$W = e^{\frac{A}{4l_p^2}} \tag{3}$$

If we plug this formula into our previous formula, we get

$$W(3) = e^{3/(4l_p^2)} = e^{(3-0.3)/(4l_p^2)} + e^{(3-0.4)/(4l_p^2)} + e^{(3-0.5)/(4l_p^2)} \tag{4}$$
$$= e^{3/(4l_p^2)} \left(e^{-0.3/(4l_p^2)} + e^{-0.4/(4l_p^2)} + e^{-0.5/(4l_p^2)} \right) \tag{5}$$

This simplifies to

$$1 = e^{-0.3/(4l_p^2)} + e^{-0.4/(4l_p^2)} + e^{-0.5/(4l_p^2)} \tag{6}$$

Of course, in this case, equality doesn't hold because I have used a hypothetical area spectrum. If we plugged in a real area spectrum, the equality would have to hold. As I explained earlier in the previous article, I obtained $0.997\cdots$ for the right-hand side of this last formula for my newly-proposed area spectrum. We conjectured that the difference of 0.003 is due to extra dimensions, which seem to modify the area spectrum.

Let me conclude this article with some comments. In reality, there are infinitely many unit areas. Therefore, (6) should be replaced by

$$1 = e^{-A_1/(4l_p^2)} + e^{-A_2/(4l_p^2)} + e^{-A_3/(4l_p^2)} + e^{-A_4/(4l_p^2)} + \cdots \tag{7}$$

which is an infinite sum. This formula was first derived by M. Domagala, J. Lewandowski and K. Meissner in 2004. Interestingly enough, the paper by Domagala and Lewandowski and the paper by Meissner, which both derived this formula appeared on arXiv, a website on which physicists upload their papers, on the same day.

Summary

- Domagala-Lewandowski-Meissner formula must be satisfied if the black hole entropy is given by the Bekenstein-Hawking entropy formula.

- Domagala-Lewandowski-Meissner formula is given by

  $$1 = e^{-A_1/(4l_p^2)} + e^{-A_2/(4l_p^2)} + e^{-A_3/(4l_p^2)} + e^{-A_4/(4l_p^2)} + \cdots$$

  where $A_1, A_2, A_3, A_4 \cdots$ are unit areas.