The free fall

Suppose you have a ball and gently release it. What happens? It falls down. Actually, everything falls down once released.

We also know that heavy objects fall faster than light objects. But, the reason why it is so is because of air resistance. Relative to its weight, the heavy objects receive less air resistance. In case air resistance is negligible, they fall at the same rate. This is what is called “free fall.” In fact, astronauts who made it into the Moon showed that a feather and a hammer fell at the same rate once released. It was so because the air is very rare in the Moon.

We also know that the further an object falls the faster its velocity. This may suggest that the velocity is proportional to the distance the object fell. That’s what Galileo first thought. However, this was wrong. Nevertheless, he later found out the correct solution: the velocity is proportional to the falling time it took to fall. In other words, we can write this as:

$$v = gt \tag{1}$$

where v is the velocity, t is the falling time, and g is their proportionality constant often called “gravitational acceleration.” It is approximately given by 9.8 m/s^2. For example, during a second of its free falling, an object gains extra 9.8 m/s of velocity downwards.

Given this, if you have read my earlier article “Newton’s second law,” the rephrased statement that the acceleration of an object free falling is roughly 9.8m/s^2 downwards should make sense. In other words, $\vec{a} = -g\hat{k}$, where $\hat{k}$ denotes the upward direction, which in turn implies $-\hat{k}$ is downward direction. Also, since we have $\vec{F} = m\vec{a}$ from our last article, we have $\vec{F} = -mg\hat{k}$, where $\vec{F}$ here denotes the gravitational force. In other words, the bigger the mass of the object the bigger the gravitational force. On the other hand, since the acceleration is inversely proportional to the mass, the acceleration of the object free falling is independent of the mass as explained earlier.

Problem 1. A grain of sand weighs 4.4 mg, i.e., 4.4×10^{-6} kg. On the Earth, what is the gravitational force (called “weight”) exerted on the grain of sand? Use g =9.8 m/s^2.

Now, let me ask another question. How far does an object free fall during first 3 seconds? The initial velocity (i.e. when released) of the object is 0m/s. After 3 seconds, it is 3 s×9.8 m/s^2=29.4 m/s. As the velocity increased at a constant rate, the average velocity for the first 3 seconds is given by the average of the initial velocity and the final velocity. Therefore, it is (0+29.4 m/s)/2=14.7 m/s. It fell for three seconds, so the total distance is given by the average velocity multiplied by three seconds, which is 14.7 m/s×3s=44.1 m.

Now, let's generalize it. After t seconds of falling, the velocity of the falling object is gt. During the t seconds, the average velocity is $(gt+0)/2 = gt/2$ and the total falling distance is given by $t \times gt/2 = (gt^2)/2$. In conclusion, the distance it fell during t seconds is given by:

$$s = \frac{1}{2}gt^2 \tag{2}$$

Given this, let me ask you another question. How much does the object fall between the time $t = t_1$ and $t = t_2$? When $t = t_1$ the velocity is given by gt_1. When $t = t_2$ it is given by gt_2. Therefore, the average velocity during the interval concerned is given by $g(t_1 + t_2)/2$. As the interval is $(t_2 - t_1)$ seconds, the total distance is given by:

$$(t_2 - t_1)\frac{g(t_1 + t_2)}{2} = \frac{1}{2}g(t_2^2 - t_1^2) \tag{3}$$

Actually, there is another way of deriving this. During first t_1 seconds it fell $\frac{1}{2}gt_1^2$. During first t_2 seconds $\frac{1}{2}gt_2^2$. Therefore, during the interval concerned, the object fell by:

$$\frac{1}{2}gt_2^2 - \frac{1}{2}gt_1^2 = \frac{1}{2}g(t_2^2 - t_1^2) \tag{4}$$

which agrees with our earlier result.

Problem 1. Suppose you release an object. Ignoring the air friction, approximately how much time does it take to fall first 1 meter?

Summary

- If you freely drop an object, its velocity is given by $v = gt$ where g is the gravitational acceleration, and t is the falling time.
- If you freely drop an object, its falling distance is given by $s = \dfrac{1}{2}gt^2$.