Gauss's law expressed using divergence

Using Stoke's theorem, we can re-express Gauss's law as follows

$$\int\int_{\partial\Omega} E \cdot dA = \int\int\int_{\Omega} (\nabla \cdot E)\, dV = \frac{q}{\epsilon_0} = \frac{1}{\epsilon_0}\int\int\int_{\Omega} \rho dV \tag{1}$$

where ρ is the charge density (i.e. electric charge per volume). Since the above equation is satisfied for any Gaussian surface or any volume region, we conclude:

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \tag{2}$$

What would be the corresponding equation for magnetic field? I want to remark that no magnetic monopole is yet found. Let me explain what it means. In case of electric charge, we have positive charges and negative charges, but in case of magnetic charges, no such thing was found. You cannot isolate a north pole of magnet or a south pole of magnet. If you cut a magnet to two pieces, you have two magnets with north and south poles, not a north pole only magnet and a south pole magnet. Therefore, we have:

$$\oint B \cdot dA = 0 \tag{3}$$

which implies:

$$\nabla \cdot B = 0 \tag{4}$$

Summary

- $\nabla \cdot E = \frac{\rho}{\epsilon_0}$
- $\nabla \cdot B = 0$