Relativistically moving charges around a square loop, an International Physics Olympiad problem

In a square loop with a side length L, a large number of balls of negligible radius and each with a charge q are moving at a speed u with a constant separation a between them, as seen from a frame of reference that is fixed with respect to the loop. The balls are arranged on the loop like the beads on a necklace, L being much greater than a, as indicated in the Figure 2.1. The no conducting wire forming the loop has a homogeneous charge density per unit length in the in the frame of the loop. Its total charge is equal and opposite to the total charge of the balls in that frame. Consider the situation in which the loop moves with velocity v parallel to its side AB (Fig. 2.1) through a homogeneous electric field of strength E which is perpendicular to the loop velocity and makes an angle θ with the plane of the loop.

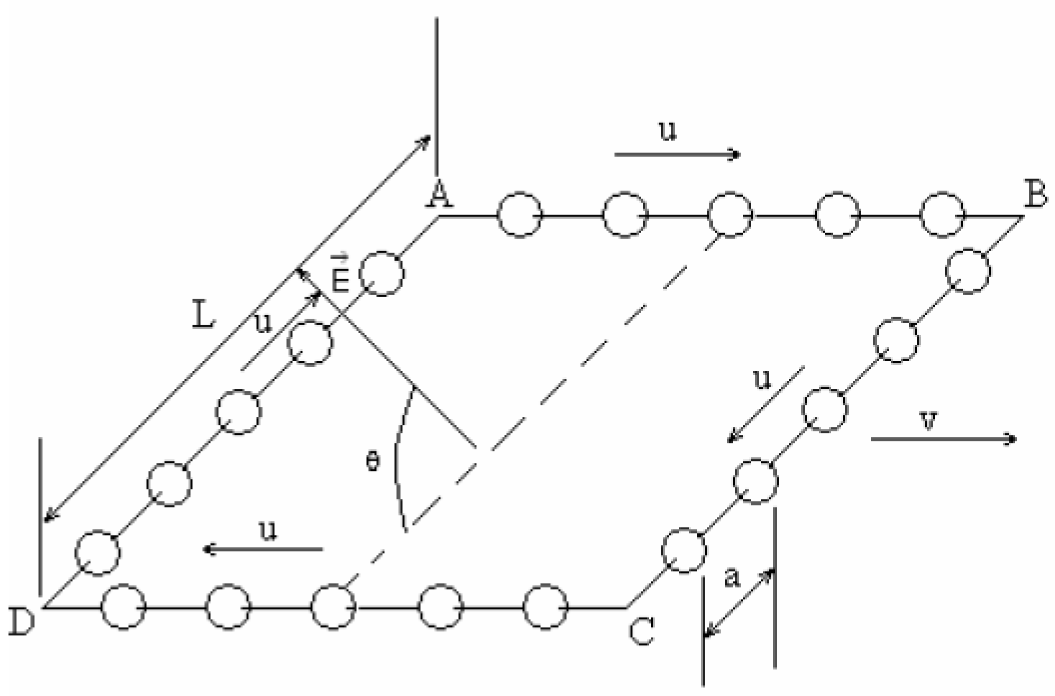

Figure 2.1

Taking into account relativistic effects, calculate the following magnitudes in the frame of reference of an observer who sees the loop moving with velocity v:

a) The spacing between the balls on each of the side of the loop, a_{AB}, a_{BC}, a_{CD}, and a_{DA}.

b) The value of the net charge of the loop plus balls on each of the side of the loop: Q_{AB}, Q_{BC}, Q_{CD}, and Q_{DA}.
c)The electrically produced torque τ tending to rotate the system of the loop and the balls.
d)The energy W due to the interaction of the system, consisting of the loop and the balls with the electric field.

All the answers should be given in terms of quantities specified in the problem.
Note. The electric charge of an isolated object is independent of the frame of reference in which the measurements takes place. Any electromagnetic radiation effects should be ignored.

Some formulas of special relativity
Consider a reference frame S' moving with velocity V with reference to another reference frame S. The axes of the frames are parallel, and their origins coincide at $t = 0$. V is directed along the positive direction of the x axis.

Relativistic sum of velocities
If a particle is moving with velocity u' in the x' direction, as measured in S', the velocity of the particle measured in S is given by:

$$u = \frac{u' + V}{1 + \frac{u'v}{c^2}}$$

Relativistic Contraction
If an object at rest in frame S has length L_0 in the x-direction, an observer in frame S' (moving at velocity V in the x-direction will measure its length to be:

$$L = L_0\sqrt{1 - v^2/c^2}$$