Lorentz force

In our earlier article "Electric field," we learned its following definition.

$$\vec{E} = \frac{\vec{F}}{q} \tag{1}$$

where $\vec{F}$ is the electric force. In other words, in the presence of the electric force, a point charge q receives the following force.

$$\vec{F} = q\vec{E} \tag{2}$$

Also, in our earlier article "Magnet exerts force on wire through which electric current passes" we learned that electric current receives magnetic force in the presence of magnetic field. Since, an electric current is charge moving, we can as well say that a charge moving receives magnetic force in the presence of magnetic field. In that article, we saw that the magnetic force is perpendicular to both electric current and magnetic field, and vanishes when the electric current and the magnetic field are parallel. This suggests that the magnetic force should be expressed using cross product between electric current and magnetic field. Moreover, the bigger electric current and the magnetic field, the bigger magnetic field. As the electric current is bigger when the electric charge and its velocity are bigger, for the magnetic force, we should have something like following.

$$\vec{F}_{\text{mag}} = k'q\vec{v} \times \vec{B} \tag{3}$$

where $\vec{v}$ is the velocity of the charge, k' a proportionality constant, and the careful consideration of the direction of magnetic force as stated earlier article on magnetic force led to $\vec{v} \times B$ rather than $\vec{B} \times \vec{v}$. However, we can set $k' = 1$ by normalizing the magnitude of $\vec{B}$. In other words, if we had the old magnetic field that satisfies $\vec{F}_{\text{mag}} = k'q\vec{v} \times \vec{B}_{\text{old}}$, then define the new magnetic field by $\vec{B}_{\text{new}} = k'\vec{B}_{\text{old}}$. This allows us to write $\vec{F}_{\text{mag}} = q\vec{v} \times \vec{B}_{\text{new}}$. In conclusion, the total electric and magnetic force exerted on an electric charge q is given as follows:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{4}$$

This is called "Lorentz force."

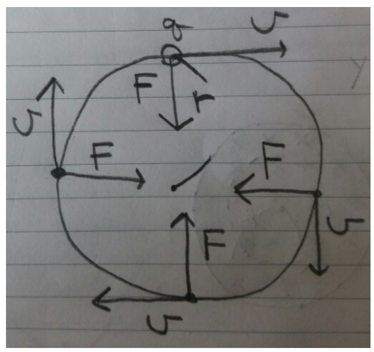
Problem 1. Can a magnetic force on moving charge ever do work? Argue why the magnetic force cannot change the speed of the moving charge.

Now, let's say that there is a uniform magnetic field $\vec{B} = B\hat{z}$ (i.e. along z direction) and there is an object with charge q moving on the $x - y$ plane with initial speed v. See the figure below. The $x-y$ plane is the plane that this page lies, and z axis is perpendicular to this page. Then, the object will receive a magnetic force perpendicular to its velocity. The magnitude of the magnetic force will be qvB. As the object will not change its speed, the magnitude of the magnetic force, which is given by qvB, won't change as well. However, the direction of the object *will* change as there is a force, and as the force and the speed don't change, it will move around in circle. The centripetal force is given by the magnetic force. Thus, we have

$$\frac{mv^2}{r} = qvB \tag{5}$$

where r is the radius of the circle. Thus, we have

$$r = \frac{mv}{qB} \tag{6}$$



Problem 2. Show that the period (i.e. the time that the object takes to complete one rotation) is given by

$$T = \frac{2\pi m}{qB} \tag{7}$$

Interestingly enough, it is independent of the speed of the object and only depends on the mass. (However, we will see later that the mass *does* change as the speed increases if the speed is not negligible compared to the speed of light.)

Problem 3. What would be the motion of the object if its initial velocity did not lie in $x-y$ plane, but were parallel to z axis? Let's say that the initial velocity was $\vec{v} = v_z\hat{z}$. (Hint[1])

Problem 4. Explain why the trajectory of the object would be helical, if its initial velocity were given by $\vec{v} = v_x\hat{x} + v_z\hat{z}$. (Hint[2])

Summary

- Lorentz force is given by

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

[1]Show that there is no magnetic force in this case.

[2]The motion would be the combination of Problem 2 and Problem 1.