Newton's second law

In the last article, we learned that an object's velocity doesn't change if no external force is exerted. Then, one may ask, what happens if an external force is exerted. Of course, it is easy to answer: the object's velocity changes. But, we need to be precise. How much does the velocity change? This is what Newton's second law answers.

To this end, let me explain what acceleration is. It is the amount of change of velocity per unit time. If you are not sure what I mean by this, let me state that velocity is the amount of change of position per unit time. If your position changes 100 meters per second, your velocity is 100 m/s. Likewise, if your velocity changes 10 m/s per second, your acceleration is 10 m/s^2.

Problem 1. If an object is moving with the constant velocity of 6 m/s, how much of its position will change during 3 seconds?

Problem 2. If an object is accelerating with the constant acceleration of 6 m/s^2, how much of its velocity will change during 4 seconds?

At this point, it is necessary to remind you the fact that velocity is a vector. Therefore, it has both a magnitude and a direction. On the other hand, speed is a scalar. It only has magnitude. Therefore, if a car moves at a constant speed but changes direction, the velocity changes. So, in such a case, the acceleration is non-zero.

Using the concept of acceleration, now we can re-phrase our earlier statement in the beginning of this article by saying that an object's acceleration is non-zero, if a non-zero force is exerted on the object. Also, we know that the more force is exerted on an object, the bigger the acceleration. We also know that if the same force is exerted on a heavy object and a light object, the heavier object budges less. In other words, the acceleration of the object is smaller if the mass is bigger.

All this suggests that the acceleration of an object is proportional to the force exerted on the object and inversely proportional to the mass of the object. This suggests that we can write the following equation:

$$\vec{a} = \frac{\vec{F}}{m} \tag{1}$$

where $\vec{a}$ is the acceleration, $\vec{F}$ the force, and m the mass. This is called Newton's second law. We notice also here that the acceleration and the force are vectors, while the mass is a scalar. This suggests that the acceleration is in the direction of the exerted force. (If you multiply or divide a vector by a scalar, only the magnitude changes while the direction remains same. See Fig.5 of "What is a vector?") This sounds natural since you expect an object, initially at rest, to move in the direction you exert a force on it.

Given the relation between acceleration, force, and mass, what is the unit for force? As force is given by mass times acceleration, the unit is given by $\mathrm{kg} \cdot \mathrm{m/s^2}$. Of course, it would be cumbersome to write this unit. Therefore, physicists introduced a unit for force denoted as "N" and pronounced as "Newton." For example, 10 N is $10\ \mathrm{kg} \cdot \mathrm{m/s^2}$.

Let me give you an example which Newton's second law implies. Let's say that you are on a train moving at constant velocity and you drop an eraser. People who haven't learned any physics could think that the eraser would drop backward, as train moves forward while it is falling. However, this is not true. The horizontal velocity of the eraser, when it is released from your hand, is same as the velocity of the train. When it falls down, only the vertical component of the velocity of the eraser changes, as the only force exerted on the eraser, that is the gravitational force, is vertical. Therefore, its horizontal velocity remains the same, as no horizontal force is exerted. This implies that the eraser moves along with the train as it falls down, which means that it drops right below the spot you released it, on the floor of the train.

Actually, this is easy to see, if you have taken an airplane, as all our earlier arguments can be equally well applied to this case. An airplane moves very fast, but if you drop an object on an airplane, it drops not backward but right below. If the law of nature were such that it should drop backward, stewardesses would have hard time pouring coffee into your cup, as the coffee would drop backward with enormous speed.

These examples can be analyzed from the point of view introduced in our last article as well. Both a train and an airplane moving at constant velocities are good examples of inertial reference frame. As the laws of physics are faithfully well applied to any inertial reference frame the eraser and the coffee would drop right downward in a train or an airplane moving at constant velocities as much as they would do on ground.

Finally, let me conclude this article by recapping my point that the acceleration is non-zero, as long as the velocity changes even though the speed remains the same. For example, let's say that you connect a string to a rock and rotate it at a constant speed by constantly pulling the string using your hand and arm. As the acceleration of the rock is non-zero, you should exert a non-zero force. Notice that the force is exerted by the tension of string, which means that the force is along the direction of string, towards your hand. Notice again that you are not exerting the force on the rock not on the direction it's moving but on the direction of the acceleration, which is towards your hand. In our later article "Centripetal force," we will calculate the amount of the force needed to rotate an object in terms of the speed of rock and the radius of its orbit. This will allow us to derive Newton's inverse square law of gravity, introduced in our earlier articles. This will be done in our later article "Newton's law of universal gravitation and Kepler's third law."

Problem 3. An object with 5 kg is initially at rest. If a total of 30 N force is exerted on it, what will be its acceleration? If this constant 30 N force is exerted on it for 4 seconds, what will be its final velocity?

Problem 4. Can an object's speed increase while its acceleration decrease?

Problem 5. Let's say a car is moving with a constant speed, but is changing its direction. Will the passengers in the car feel inertial force? If your answer is yes, toward which direction will they feel the inertial force, in the case in which the car is turning left?

Summary

- Newton's second law is given by
$$\vec{a} = \frac{\vec{F}}{m}$$
where $\vec{a}$ is the acceleration, $\vec{F}$ the force, and m the mass.