Newton's third law and the conservation of momentum

According to wikipedia, Newton's third law is stated as: "When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction to that of the first body." For example, see Fig. 1. If the girl (i.e. the first body) pushes (i.e. exerts a force) the boy (i.e. the second body), the girl will feel a force directed against her; she will move leftward.

Actually, Newton's third law is valid not only for the contact force such as in our example. Newton's third law is also valid for the force between two distant objects. For example, as much as the Sun is pulling the Earth, making it rotate, the Earth is pulling the Sun with the same amount of force. Of course, as the Sun is much heavier than the Earth, the effect of the force is negligible to the one on the Earth by the Sun.

Now, we are ready to derive the conservation of momentum. Suppose that we have two objects: object 1 and object 2. Let $\vec{F}_{21}$ be the force the object 1 exerts on the object 2 and $\vec{F}_{12}$ be the force the object 2 exerts on the object 1. If we denote the momentum of the object 1 by p_1, and similarly for the object 2, we have:

$$\Delta\vec{p}_1 = \Delta(m\vec{v}_1) = m\Delta\vec{v}_1 = m\frac{\Delta\vec{v}_1}{\Delta t}\Delta t = \vec{F}_{12}\Delta t \tag{1}$$

Similarly,

$$\Delta\vec{p}_2 = F_{21}\Delta t \tag{2}$$

By Newton's third law, we have $\vec{F}_{21} = -\vec{F}_{12}$. Using this, we obtain:

$$0 = (\vec{F}_{12} + \vec{F}_{21})\Delta t = \Delta(\vec{p}_1) + \Delta(\vec{p}_2) = \Delta(\vec{p}_1 + \vec{p}_2) \tag{3}$$

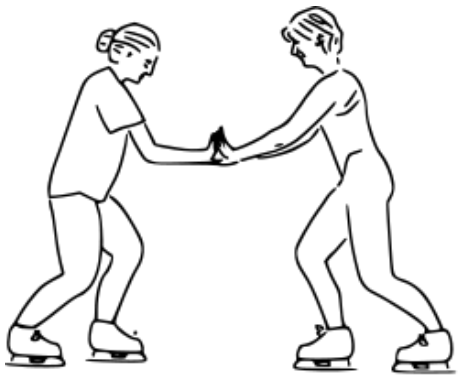

Figure 1: Newton's third law

This implies that the sum inside parenthesis never changes. Therefore, we conclude:

$$p_1 + p_2 = \text{constant} \tag{4}$$

In other words, we say that the momentum is conserved.

Problem 1. If an apple falls from a tree, it gains momentum downward. What happens to the momentum conservation? Is it violated?

Problem 2. Let's say that an object with mass 1 kg and velocity 3 m/s collides with an object at rest with mass 2 kg and they stick together. What would be the velocity of the combined object? How much kinetic energy is lost during the collision? Collisions like this, in which the kinetic energy is lost through heat and sound are called "inelastic collisions." In the next two articles we will consider elastic collisions in which the kinetic energy is not lost.

Summary

- Newton's third law is: "When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction to that of the first body."
- Newton's third law is responsible for the conservation of momentum.

(The figure is from `http://en.wikipedia.org/wiki/File:Skaters_showing_newtons_third_law.svg`)