Tully-Fisher relation, Modified Newtonian Dynamics, and Verlinde gravity

In our third article on history of astronomy, I briefly explained Tully-Fisher relation and MOND (Modified Newtonian Dynamics). In this article, I will explain the actual formulas. Tully-Fisher relation says that v, the speed of the outermost stars orbiting around the center of galaxy is proportional to the square root of the square root of M, the (visible) mass of galaxy, and is independent of the distance from the center of galaxy as long as the distance is large enough. In other words,

$$v = bM^{1/4} \tag{1}$$

where b is a certain constant.

Let's see why the Newtonian gravity can't explain this without assuming the dark matter. In other words, if we assume only the mass of the visible matter contributes to the gravity. Remember that the centripetal force of the star is provided by the gravitation of the galaxy. If we make an approximation that all the (visible) mass of the galaxy is at its center, we have

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \tag{2}$$

Then, we get

$$v = \sqrt{\frac{GM}{r}} \tag{3}$$

which is totally different from (1), which doesn't depend on r. Moreover, v is proportional to $M^{1/2}$ not to $M^{1/4}$ as in Tully-Fisher relation.

In 1983, as an alternative to dark matter theory, the Israeli physicist, Mordehai Milgrom tried to derive Tully-Fisher relation by modifying Newton's second law, as follows.

$$F = ma \quad (\text{for} \;\; a \gg a_M), \qquad F = m\frac{a^2}{a_M} \quad (\text{for} \;\; a \ll a_M) \tag{4}$$

where a_M is Milgrom's constant, known to be around $1.2 \times 10^{-10}\text{m/s}^2$. The first condition must be satisfied, because Modified Newtonian Dynamics must be reduced to Newton's second law in our daily lives. From numerous experiments, we know that Newton's second law is correct when the acceleration is not that extremely small.[1] (i.e., $a \gg a_M$) However, Milgrom

[1] Of course, assuming the speed of the object is much smaller than the speed of light. Otherwise, we need to consider the relativistic effect.

noted that Newton's second law had never been tested for such a law acceleration regime (i.e., $a \ll a_M$), which makes such a modification of Newtonian dynamics not implausible. His proposal of (4) is called MOND (Modified Newtonian Dynamics).

Now, let's see how this modification explains Tully-Fisher relation. First, note that the centrepital acceleration of a star orbiting around the center of galaxy with speed v and with orbit radius r is given by

$$a = \frac{v^2}{r} \tag{5}$$

The orbital rotation of stars around the center of galaxy correspond to very low acceleration regime (i.e., $a \ll a_M$). Thus, instead of (2), we now have

$$m\frac{a^2}{a_M} = m\frac{(v^2/r)^2}{a_M} = \frac{GMm}{r^2} \tag{6}$$

from which we get

$$v = (GMa_M)^{1/4} \tag{7}$$

which is exactly (1). Milgrom found the value of a_M from the value of b in (1).

Then, you may wonder what Newton's second law should look like when neither of the conditions in (4) (i.e., $a \gg a_M$ and $a \ll a_M$) is satisfied. Let's look at (4) more closely. If we write the Modified Newton's second law as

$$F = m\mu\left(\frac{a}{a_M}\right)a \tag{8}$$

we need to have

$$\mu(x) = 1 \quad (\text{for} \quad x \gg 1), \qquad \mu(x) = x \quad (\text{for} \quad x \ll 1) \tag{9}$$

Of course, there are many functions that satisfy the above criteria. But, Milgrom's theory doesn't say which one of them is the one that God actually chose. So, many people tried different $\mu(x)$ and fit them with the galaxy rotation curves. For examples, functions such as

$$\mu(x) = \frac{1}{1+\frac{1}{x}}, \qquad \mu(x) = \sqrt{\frac{1}{1+(\frac{1}{x})^2}} \tag{10}$$

satisfy the above criteria.

At this point, we would like to mention that the original Tully-Fisher relation was not a relation between the rotation speed of outermost stars and the mass of the galaxy, strictly speaking. The title of the original paper [1] by R. Brent Tully and J. Richard Fisher, submitted in 1975 and published in 1977 was "A New Method of Determining Distances to Galaxies," which may sound puzzling to you, who just read my presentation of Tully-Fisher relation. What they reported in their paper was the relation between the emission line widths and the absolute magnitudes (i.e., absolute brightness) of galaxies.

What is the emission line width? Recall how we can determine the receding speed of a galaxy. As we explained in "Doppler effect: How can we know the speed of stars?" if we measure the emission line of a galaxy, i.e., the wavelengths of light emitted from the atoms in the galaxy, we can determine the receding (or approaching) speed of galaxy from the red-shift (or blue-shift) of the emission line of the galaxy.

However, do all the stars in the same galaxy have the same receding speed from us? No. As the stars are rotating around the center of the galaxy, some will recede (move away) from us faster, and others will recede from us slower, even though they are all located in the same galaxy. Therefore, the receding speed of stars in a galaxy does not have a single value, but a certain range of values depending on how fast the stars are rotating in the galaxy. The bigger the maximum rotation speed of stars in a galaxy the bigger the range of their receding speed from us. Therefore, if we measure the emission line, the wavelength won't be single-valued, but will have some width. This is the emission line width.

Then, what is the absolute magnitude of a galaxy? It is how much light a galaxy is emitting, or how bright a galaxy is, if we compensate the effect that the farther we are from the galaxy, the fainter the galaxy seems. If we are farther from a galaxy, the apparent magnitude is smaller, but the absolute magnitude doesn't change.

Now, let's connect. The emission line width is related to the maximum rotation speed of stars in a galaxy. In our case, v on the left-hand side of (1). The absolute magnitude of a galaxy is related to its mass; the brighter a galaxy is, the more stars it has, and the heavier it is. Thus, it is related to M on the right-hand side of (1). Therefore, the original Tully-Fisher relation is directly related to the modern form of the Tully-Fisher, explained in this article.

Let's return now to the original title of Tully's and Fisher's paper. Assuming that Tully-Fisher relation is correct, if we measure the emission line width, we can determine the absolute brightness of a galaxy. Once the absolute brightness of a galaxy is determined, we can measure the distance to it by measuring its apparent brightness; the smaller the apparent brightness, the farther the galaxy is from us.

Coming back to our main discussion, I want to point out a remarkable paper [2] at this point. Tully and Fisher related the mass of stars in a galaxy with the maximum rotation speed of stars in a galaxy. However, stars are not the only things in galaxies, even when we disregard dark matter, which do not exist in dark matter-less theory; we know that gas is spread inside galaxies. Therefore, the total mass of a galaxy does not only include the stars in it, but also the gas in it, even though the gas does not emit visible light, which therefore is neglected in the original Tully-Fisher relation. In other words, not only the stars, but also the gas contribute to the gravitation. Therefore, M in (6) must include not only the mass of the stars but also the mass of gas. Accordingly, M in (7) now includes the mass of gas in addition to the mass of stars.

Tully-Fisher relation, (7), worked, because the mass of gas was not significant compared to the mass of stars. However, if Modified Newtonian Dynamics is correct, one expects that Tully-Fisher relation must work better, if M includes the mass of gas.

The authors of [2] exactly checked this. See Fig. 1. We see that the right figure fits better than the left figure. They called their new findings "baryonic Tully-Fisher relation." Protons and neutrons are types of baryons. Therefore, nuclei of atoms are baryons. Physicists and astronomers know that dark matter must not be baryons, if it exists. Therefore, in astronomy literature, the term "baryon" is used to denote ordinary matter, such as atoms, that are not dark matter. Even though atoms have electrons which are not strictly baryon according to the definition of physicists, the use of the term baryon in astronomy is justified considering that the mass proportion of baryon in atoms is about 99.95%.

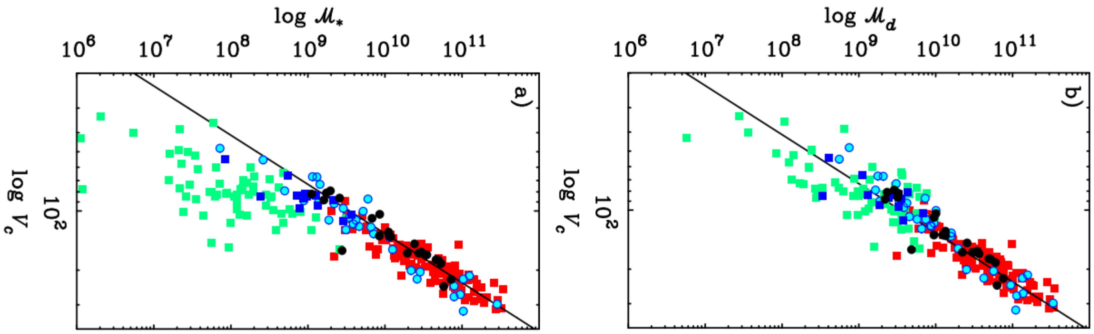

Figure 1: The left figure is the original Tully-Fisher relation with M the total mass of stars in a galaxy. The right figure is the baryonic Tully-Fisher relation with M the mass of stars and gas altogether in a galaxy. The different colors denote the data from different sources. Figures reproduced by permission of the AAS [2].

Enough with Modified Newtonian Dynamics. Let's turn to Verlinde gravity. In his original paper on Verlinde gravity, Verlinde derived Tully-Fisher relation. When I first read it, I was very surprised by this. However, his theory is not MOND; instead of modifying Newton's second law, Verlinde suggested a theory of gravitation that is different from Newton's or Einstein's. What was also remarkable was that he correctly derived Milgrom's constant. He obtained

$$a_M = \frac{cH_0}{6} \tag{11}$$

where c is the speed of light, and H_0 is Hubble's constant.

Problem 1. Find c and H_0 from Internet, and calculate the value of Milgrom's constant Verlinde obtained. Then, compare with Milgrom's constant $a_M \equiv 1.2 \times 10^{-10}\text{m/s}^2$. If you correctly solve this problem, the values will agree within about 10 % of error.

In our article "Verlinde gravity and galaxy rotation curves," I will explain the MOND fitting of galaxy rotation curve, and how Verlinde gravity outperforms it.

Summary

- Tully-Fisher relation says that the rotation speeds of outer most stars in a galaxy do not depend on the distance from the center of galaxy, and scale as the square root of the square root of the total mass of galaxy, contrary to the Newtonian prediction.
- Tully-Fisher relation fits better if mass of the gas is included in the galaxy mass, instead of just the mass of stars. This is known as "baryonic Tully-Fisher relation."
- Milgrom suggested a modification of Newton's second law for very low acceleration to explain Tully-Fisher relation. This is called "MOND" (Modified Newtonian Dynamics). In particular, he introduced "Milgrom's constant" which sets the scale of MOND.
- Verlinde correctly derived Milgrom's constant and Tully-Fisher relation from his theory of gravity. Nevertheless, his theory is not MOND, as he didn't modify Newton's second law; he suggested a theory of gravity that is different from Newton's or Einstein's.

References

[1] R. B. Tully and J. R. Fisher, "A New method of determining distances to galaxies," Astron. Astrophys. **54**, 661-673 (1977)

[2] S. S. McGaugh, J. M. Schombert, G. D. Bothun and W. J. G. de Blok, "The Baryonic Tully-Fisher relation," Astrophys. J. Lett. **533**, L99-L102 (2000) doi:10.1086/312628 [arXiv:astro-ph/0003001 [astro-ph]]. `https://iopscience.iop.org/article/10.1086/312628`