Big Bang nucleosynthesis

Those of you who closely follow popular science may know that our Universe is 75% hydrogen, and 25% helium (by mass fraction). Actually, we know this, thanks to observations of astronomers. What is remarkable is that theoreticians can predict this number from the standard big bang cosmology. We will sketch how we can obtain this number. In the last article, we talked about the period when the nuclei already formed combined with electrons to form atoms. In this article, we go further back to the period when these nuclei were first formed.

In the early universe, long before first nuclei were formed, neutrons and protons had been in thermal equilibrium by following reactions due to weak interactions:

$$n + \nu_e \quad \leftrightarrow \quad p + e^- \tag{1}$$

$$n + e^+ \quad \leftrightarrow \quad p + \bar{\nu}_e \tag{2}$$

However, their reaction rates began to fall behind the expansion rate of our Universe as the temperature of our Universe dropped below 0.8 MeV (i.e., when our Universe was around 1 sec old). Neutrons were no longer in a thermal equilibrium with proton. As the reactions that convert neutron to proton and proton to neutron became rare, the neutron to proton ratio freezed out (i.e., became fixed). Let's calculate this ratio.

Neutron mass is about 939.565 MeV and proton mass is about 938.272 MeV. As they are much bigger than their temperature 0.8 MeV, we can safely treat them non-relativistically. Thus, considering that $\mu_n = \mu_p$ as the chemical potential for electrons and neutrinos are negligible, we have

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-(m_n - m_p)/T} \tag{3}$$

Here, we can make the approximation $m_n \approx m_p$ for the ratio in the parenthesis, but not in the exponent. Then, we have

$$\frac{n_n}{n_p} = e^{-(m_n - m_p)/T} \tag{4}$$

$m_n - m_p$ is about 1.293 MeV, which is slightly bigger than the temperature 0.8 MeV. Thus, we expect that n_n is not very very small compared to n_p. If we plug in these values, we get $n_n \approx 0.2 n_p$. The total number of baryons is given by $n_B = n_n + n_p$. Thus, the proportion that neutron has among baryons at this moment is about

$$\frac{n_n}{n_B} \approx \frac{1}{6} \tag{5}$$

Of course, the freeze-out doesn't happen instantly, as the reaction doesn't suddenly stop when the temperature cooled down to 0.8 MeV. Anyhow, a full analysis yields a close value.

However, the freeze-out is only a thermal one and temporary as the neutron fraction changes further by the following beta decay that converts neutrons into protons

$$n \rightarrow p + e^{-} + \bar{\nu}_e \tag{6}$$

The mean decay lifetime is about 886 seconds. The reaction above only occurs for free neutrons. Once the neutrons combine with protons to form "deuteron" (D) by following reaction:

$$n + p \rightarrow D + \gamma \tag{7}$$

the beta decay of neutron stops, as neutrons are stable inside stable nuclei such as deuteron. You see here that a deuteron has one proton and one neutron. It is the nucleus of an atom called "deterium" which is an isotope of hydrogen.

Thus, if we calculate the age of universe t when the temperature of our Universe was low enough that the deuteron was formed, we can obtain the final neutron fraction by following formula

$$\frac{n_n}{n_B} = 0.16 e^{-(t-t_0)/886\mathrm{s}} \tag{8}$$

where t_0 is the age of universe at the thermal freeze-out of neutron and proton (i.e., $t_0 \sim 1$ sec). Of course, this is a rough estimate, as the process (7) does not happen instanteneously.

Let's closely look at the reaction (7). As the chemical potential of photon is zero, we have

$$\mu_n + \mu_p = \mu_D \tag{9}$$

Then, just as in the last article, at equilibrium, we obtain

$$\frac{n_D}{n_n n_p} = \frac{g_D}{g_n g_p} \left(\frac{m_D}{m_n m_p} \frac{2\pi}{T} \right)^{3/2} e^{B_D/T} \tag{10}$$

where $B_D = m_n + m_p - m_D = 2.22\mathrm{MeV}$ is the binding energy of deuteron.

We have $g_n = g_p = 2$, because both neutron and proton are spin 1/2 particles. On the other hand, deuteron is a spin 1 particle, so $g_D = 3$ (i.e., $m = -1, 0, 1$). As before, we can make the approximation that $m_D = 2m_n = 2m_p$ in the ratio in the parenethesis, while we cannot ignore $B_D = m_n + m_p - m_D = 2.22$ MeV in the exponent.

So, we want to obtain the temperature at which the following ratio becomes the order of unity or much bigger.

$$\frac{n_D}{n_n} = \frac{3}{4} n_p \left(\frac{4\pi}{m_n T} \right)^{3/2} e^{B_D/T} \tag{11}$$

That is when deuterons begin to form or most of neutrons have already combined with protons to form deuterons. We naively expect that this happens when the temperature is around 2 MeV. However, just like in the last article, due to very low baryon to photon ratio, this is significantly delayed. Let's see until when. First, we have

$$n_B = \eta n_\gamma = \eta \frac{2\zeta(3)}{\pi^2} T^3 \tag{12}$$

Thus, (11) becomes

$$\frac{n_D}{n_n} = \eta \frac{n_p}{n_B} \frac{12\zeta(3)}{\sqrt{\pi}} \left(\frac{T}{m_n}\right)^{3/2} e^{B_D/T} \tag{13}$$

We know that the ratio n_p/n_B is in the order of unity. As we have seen, at the freeze-out, it was around $1-1/6 = 5/6$. Also, if we "cheat" to see the final answer, it is around 0.75 (i.e., 75 % of n_B is the proton is the hydrogen nucleus) at the end (i.e., when the Universe is at the temperature we want to obtain). In either case, it doesn't significantly affect the temperature we want to obtain. See Fig. 1 for the ratio of $n_n/(n_n + n_D)$. When the temperature was high, most of the neutrons were free neutrons. However, as temperature dropped, suddenly they are in the form of deuterons. This was when the temperature was around 0.06 MeV, i.e., 330 second old. Plugging $t = 330$ sec into (8), we obtain

$$\frac{n_n}{n_B} \approx 0.113 \tag{14}$$

I want to remark that n_n here is not n_n in (13). n_n in (14) is the neutron number density of either in the form of free neutron or in the from of deuteron. As there are no longer free neutrons now, the neutron number density in (14) is actually the deuteron number density, i.e., n_D in (13).

I will upload Figure 1 later

What happens after deuteron is formed? Once deuteron is formed, they collide to form the helium nuclei by following reactions:

$$\begin{aligned} D + D &\rightarrow {}^3\mathrm{H} + p & (15)\\ D + D &\rightarrow {}^3\mathrm{He} + n & (16)\\ D + {}^3\mathrm{H} &\rightarrow {}^4\mathrm{He} + n & (17)\\ D + {}^3\mathrm{He} &\rightarrow {}^4\mathrm{He} + p & (18)\end{aligned}$$

Thus, deuterons, ^{3}H and ^{3}He quickly disappear by the above reactions, and neutrons are exclusively inside helium nuclei, which are very stable. Some of them further collide to form other atom nuclei, but their quantity is very small. As there are equal number of neutrons and protons in helium nuclei (i.e., two), and $m_n \approx m_p$, the mass fraction of helium in our Universe, often denoted "Y_p" must be the double of (14). Then, we get $Y_p \approx 0.226$. We got a slightly wrong value, because we assumed thermal equilibrium, but a treatment of non-equilibrium thermodynamics leads to the right value. For example, a recent study[1] says that the observed value is $Y_p = 0.2449 \pm 0.0040$ while the theoretical value is $Y_p = 0.24709 \pm 0.00017$, which is a perfect agreement.

Final comment. We saw that before deterons were formed, formation of nuclei other than the one of protons were not possible. This is known as "deuterium bottleneck." A bottle usually has a very tight neck. Once the contents inside a bottle go through the neck, then they come out easily forming helium nuclei and so on.

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[1] C. Pitrou, A. Coc, J. P. Uzan and E. Vangioni, "Precision big bang nucleosynthesis with improved Helium-4 predictions," Phys. Rept. **754**, 1 (2018) doi:10.1016/j.physrep.2018.04.005

Summary

- Big bang nucleosynthesis is a synthesis (i.e., formation through combining) of nuclei in our early Universe.
- In mass fractions, 75% of our Universe is hydrogen, and 25% of our Universe is helium. The standard big bang nucelosynthesis theory correctly predicts these values which are confirmed by observations.
- Long before first nuclei were formed, protons and neutrons were in thermal equilibrium.
- However, as temperature dropped, they were no longer in thermal equilibrium and the free neutrons began to decay.
- It continued until the free neutrons were combined with protons to form deuterons. As neutrons were now stable, the total neutron numbers (which are now inside deuteron) were fixed.
- The deuterons so formed further collide with other deuterons to form ^{3}H and ^{3}He, which again collied with deuterons to form ^{4}He. Once the helium nuclei were formed, they were very stable, and further reactions rarely happened.
- The deuterium bottleneck describes the situation that the nucleosynthesis began to happen rapidly only when the first deuterons were formed.