Capacitor

See Fig.1. Suppose a plate is located at $z = d$ where electric charge with a uniform surface charge density σ is distributed and another plate is located at $z = 0$ where electric charge with a uniform surface charge density $-\sigma$ is distributed. What will be the electric field? See Fig.2. We can calculate this, by adding up the electric field due to the first plate and the one due to the second plate. For the first one, we have:

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{z} \quad (z > d), \qquad \vec{E} = -\frac{\sigma}{2\epsilon_0}\hat{z} \quad (z < d) \tag{1}$$

For the second one, we have:

$$\vec{E} = -\frac{\sigma}{2\epsilon_0}\hat{z} \quad (z > 0), \qquad \vec{E} = \frac{\sigma}{2\epsilon_0}\hat{z} \quad (z < 0) \tag{2}$$

Adding them up, we have:

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{z} \quad (0 < z < d), \qquad \vec{E} = 0 \quad (\text{otherwise}) \tag{3}$$

Now, suppose the plate is not infinitely wide but wide enough compared to d so we can regard it approximately as infinitely wide when calculating the electric field. If the charge on upper plate is Q and the one on the lower plate is $-Q$, and each of them has area A, we have $\sigma = Q/A$. Then, the electric field on the region between the plates is given as follows:

$$\vec{E} = \frac{Q}{\epsilon_0 A} \tag{4}$$

Given this, what is V the potential difference between the two plates? It is left as an exercise to the reader to show the following:

$$V = \frac{Qd}{\epsilon_0 A} \tag{5}$$

Notice that V is proportional to Q. So, if we define C "the capacitance" as follows,

$$C = \frac{Q}{V} \tag{6}$$

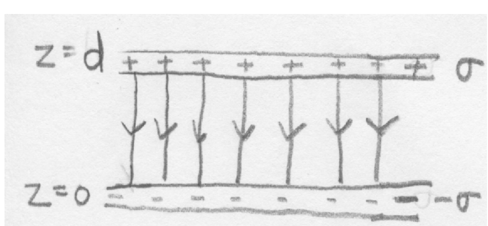

Figure 1: capacitor without dielectric

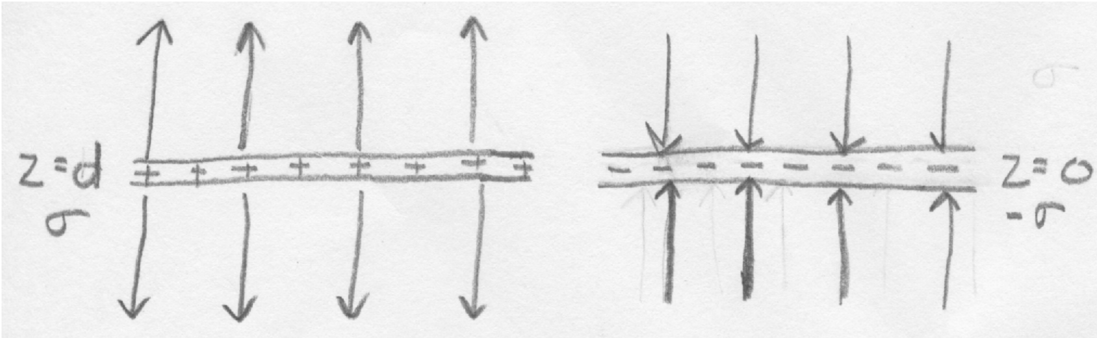

Figure 2: perspective view

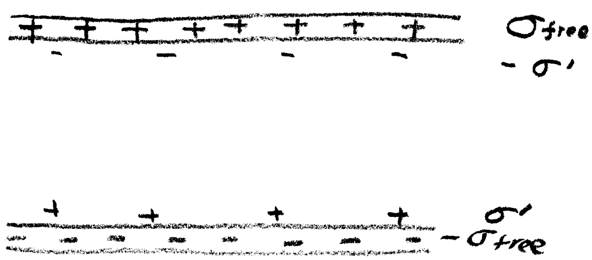

Figure 3: Capacitor with dielectric

we have:

$$C = \epsilon_0 \frac{A}{d} \tag{7}$$

Now, how will this change, if there is a medium called "dielectric" between the two plates? See Fig.3. As was the case in our earlier article "Maxwell's equations in matter," negative charge is cluttered around the positive charge on the upper plate, while positive charge is cluttered around the negative charge on the lower plate. It is easy to see that this weakens electric field. Taking the same step as in the earlier article, we obtain:

$$\vec{E} = \frac{Q}{\epsilon A} \tag{8}$$

Therefore, the capacitance is given by

$$C = \epsilon \frac{A}{d} \tag{9}$$

In any case, we can see that we can store Q this way. Capacitor, which hoards electric charge this way, is useful when giving electric shock to someone having heart attack.

Problem 1. Is the capacitance bigger or smaller in the presence of dielectric?

Problem 2. If $\sigma_{\text{free}} = Q/A$ in Fig.3, what is σ' in terms of Q, A, ϵ_0 and ϵ?

Summary

- The capacitance is defined by $C = \dfrac{Q}{V}$. Where Q is the charge and V is the potential difference between the two plates. (The bigger the capacitance, the more charge a capacitor can store charge.)