Centripetal force

Suppose an object moves along a circular orbit with constant speed. What would be its acceleration, and what would be the force for it? At first glance, the acceleration may seem to be zero, since it's moving at a constant speed. Nevertheless, this is not the case since the direction of the velocity constantly changes and this change per unit time gives the acceleration. Let's go on to calculate the acceleration.

See Fig.1. An object is moving clockwise with radius r with speed v. At a certain time, the object's velocity was $\vec{v_1}$. After a very short time Δt has elapsed, it moved by a very short distance

$$\Delta s = v \Delta t = r \Delta \theta \tag{1}$$

and the velocity has become \vec{v}_2 . (Here, we are using radian for the angle $\Delta \theta$.) Let's scrutinize how much the velocity has changed. See Fig.2. We easily see that $\Delta v \approx v \Delta \theta$, since $\Delta \theta$ is very small. Therefore, the acceleration is given as follows:

$$a = \frac{\Delta v}{\Delta t} = \frac{v\Delta\theta}{\Delta t} = v\frac{\Delta\theta}{\Delta t} = v\frac{v\Delta t/r}{\Delta t} = \frac{v^2}{r}$$
(2)

where we have used (1). Notice also that the acceleration is toward the center as evident from comparing the direction of \vec{v} in Fig.2 with the direction of center in Fig.1. Therefore, to move an object with mass m along such a circular orbit, with speed v, we need the following force toward the center:

$$F = ma = \frac{mv^2}{r} \tag{3}$$

That the force is toward the center indeed makes sense; if you attach a string to a rock and make it keep rotating using your hand and arm, you need to pull the string. (i.e. exert a force toward the center which is your hand)

Problem 1. Can centripetal force ever do work?



Figure 1: an object rotating



Summary

• To move an object with mass m along a circular orbit with radius r with speed v we need the centripetal force given by

$$F = \frac{mv^2}{r}$$