Constant acceleration in 1-dimension

Suppose you are initially (i.e. $t = 0$) at position $x = 0$m, then, move 10m/s in positive x-direction for 3.5 seconds then 20m/s in positive x-direction for 4.5 seconds. Then, where are you now? It is very easy. We have:

$$0\text{m} + 10\text{m/s} \times 3.5\text{s} + 20\text{m/s} \times 4.5\text{s} = 125\text{m} \tag{1}$$

If you draw time versus velocity graph as in Fig. 1, it is easy to see that the distance traveled is equal to the area enclosed by the graph, the axis of time, the lines $t = 0$ and $t = 8$.

Now, let's consider our main example in this article. Let's say an object moves along a straight line and starts moving with the initial velocity v_i, and accelerates with a constant value a for time t. What is the final velocity? As the velocity of the object increases a m/s every second, the total increase is at. Therefore, the final velocity is given as follows:

$$v_f = v_i + at \tag{2}$$

See Fig. 2. Given this, what is the total distance traveled? It is the area of the shaded region, which is:

$$s = \frac{(v_i + v_f)t}{2} = v_i t + \frac{1}{2}at^2 \tag{3}$$

Of course, without the graph, we could have obtained the same result using our method in "The free fall:" The mean velocity is $(v_i + v_f)/2$, so the total distance traveled is $((v_i + v_f)/2)t$. Nevertheless, it is nice to get some visualization and looks the same thing at different angles.

Finally, let's assume that we know the initial velocity, the final velocity and the acceleration. Can we obtain the total distance traveled? It is easy. First notice:

$$t = \frac{v_f - v_i}{a} \tag{4}$$

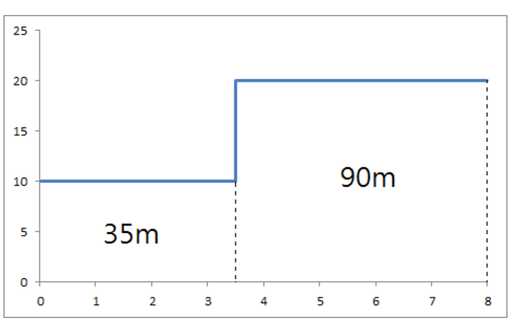

Figure 1:

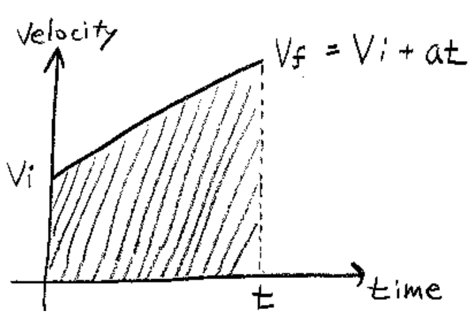

Figure 2:

Plugging this to (3), we get:

$$s = \frac{(v_i + v_f)(v_f - v_i)}{2a} = \frac{v_f^2 - v_i^2}{2a} \tag{5}$$

This equation turns out to be useful, when we will talk about kinetic energy.

Problem 1. On a straight road, a car, initially at rest, accelerates at a constant rate. After traveling 100 m, it reaches the speed 30 m/s. What is its acceleration?

Summary

- For a constant acceleration a, if the initial velocity was v_i, the final velocity after time t is given by

$$v_f = v_i + at$$

- The distance traveled during such time t is given by

$$s = \frac{v_i + v_f}{2} t$$