The determinant of product of two matrices

In this article, we will prove that the determinant of product of two matrices is given by the product of determinant of each matrix. In other words,

$$\det(AB) = \det A \det B \tag{1}$$

To prove this, let's define a function of A as follows:

$$f(A) = \frac{\det(AB)}{\det B} \tag{2}$$

Now, we can easily check that the above function satisfies the three properties of the determinant. Since such a function is unique, it has to be determinant. Therefore, we conclude

$$\det A = \frac{\det(AB)}{\det B} \tag{3}$$

which implies (1). This concludes the proof.

Problem 1. Check that (2) indeed satisfies these properties.

Problem 2. Prove the following:

$$\det(A^{-1}) = \frac{1}{\det A} \tag{4}$$

Argue why the inverse matrix A^{-1} doesn't exist, if $\det A = 0$.

Problem 3. Prove the following:

$$\det(ABC) = \det A \det B \det C \tag{5}$$

Problem 4. What can you say about the determinant of a unitary matrix?

Summary

- $\det(AB) = \det A \det B$