The dot product and law of cosines

In this article, using law of cosines, we will derive

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta \tag{1}$$

where θ is the angle between $\vec{A}$ and $\vec{B}$.

Given this, see Fig.1. We clearly have $\vec{C} = \vec{A} - \vec{B}$, and the law of cosines state the following:

$$|\vec{C}|^2 = |\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}| \cos\theta \tag{2}$$

as $|\vec{A}|$, $|\vec{B}|$, $|\vec{C}|$ are the lengths of sides of the triangle.

Furthermore, as we know that

$$\begin{aligned} |\vec{C}|^2 &= \vec{C} \cdot \vec{C} = (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B}) = \vec{A} \cdot \vec{A} + \vec{B} \cdot \vec{B} - 2\vec{A} \cdot \vec{B} && (3) \\ &= |\vec{A}|^2 + |\vec{B}|^2 - 2\vec{A} \cdot \vec{B} && (4) \end{aligned}$$

Comparing with (2) we obtain (1).

Problem 1. Show that the magnitudes of $\vec{D}$ and $\vec{E}$ are same, if $\vec{D} + \vec{E}$ and $\vec{D} - \vec{E}$ are perpendicular. (Hint[1])

Problem 2. Calculate the angle between $\vec{F}$ and $\vec{G}$ if $|\vec{F}| = |\vec{G}|$ and $|\vec{F} + \vec{G}| = |\vec{F} - \vec{G}|$. (Hint[2])

[1] If two vectors are perpendicular, their dot product is zero.

[2] Use $|\vec{F}|^2 = |\vec{G}|^2$ and $|\vec{F} + \vec{G}|^2 = |\vec{F} - \vec{G}|^2$.

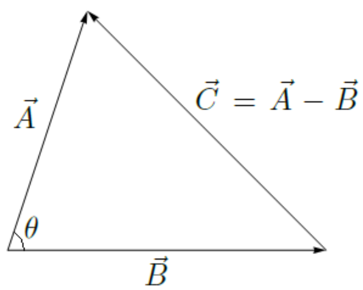

Figure 1: A triangle formed by $\vec{A}$, $\vec{B}$, and $\vec{C}$