Why was very early universe dominated with light?

In our earlier article, we have learned that our universe is expanding. Then, the density for matter and light will decrease accordingly. For ordinary matter, the density is inversely proportional to the volume of universe. For light, the density does decrease as the universe expands, but the density is not exactly inversely proportional to the volume. In this article, we will find the exact relation.

Remember our earlier formula in "Specific heat of gases":

$$\Delta Q = P\Delta V + \Delta U \tag{1}$$

Since the universe is expanding without any significant energy heated to it, we have $\Delta Q = 0$. Now using $P = \rho/3$, and denote the volume of our universe by V, we have:

$$\begin{aligned} 0 &= \frac{\rho}{3}\Delta V + \Delta(\rho V) \\ &= \frac{\rho}{3}\Delta V + \Delta\rho V + \rho\Delta V = \frac{4}{3}\rho\Delta v + \Delta\rho V \\ 0 &= \frac{4}{3}\frac{\Delta V}{V} + \frac{\Delta\rho}{\rho} \\ &= \frac{4}{3}\Delta(\ln V) + \Delta(\ln\rho) = \Delta(\frac{4}{3}\ln V + \ln\rho) \\ &= \Delta(\ln(V^{4/3}\rho)) \end{aligned} \tag{2}$$

So, we conclude that $V^{4/3}\rho$ is constant. In other words, the photon energy density ρ is inversely proportional to $V^{4/3}$. Compare this with the ordinary matter in which the energy density is inversely proportional to V. In cosmology, it is customary to call ordinary matter density by the matter density and light energy density by the radiation density and denote the linear size of universe by a, which we call the "scale factor." As V is proportional to a^3, the matter density is proportional to $1/a^3$ while the radiation density is proportional to $1/a^4$.

In present, there are much more matter than the light in our universe. We call such a universe "matter-dominated universe." However, as the radiation density has decreased faster than the matter density, we can conclude that there was a time when the radiation density was bigger than the matter density. We call such a universe "radiation-dominated universe."

Two comments. First, there is another way of seeing that the radiation density decreases as $1/a^4$. As universe expands, the wavelength of each photon expands accordingly. In other words, the wavelength of each photon is proportional to a. We have already talked about it when we talked about "red-shift" in our article "The expanding universe." As the energy of a photon is inversely proportional to the wavelength, the energy of a photon is proportional to $1/a$. As the number density of photons is inversely proportional to the volume, the former is proportional to $1/a^3$. Multiplying $1/a^3$ by $1/a$ we conclude the energy density of photon is proportional to $1/a^4$.

Second, by using (1) we can derive the familiar fact that the energy density of ordinary matter is inversely proportional to V as we did for analogous result for the radiation. To this end, recall

$$0 = P\Delta V + \Delta(\rho V) \tag{3}$$

where ρ and p are now the energy density and the pressure of ordinary matter. However, at first glance, this formula may not make much sense; as we know that our conclusion must be ρV is constant for ordinary matter, we must have $P = 0$ for ordinary matter. On the other hand, we know that the pressure of ordinary matter is not generally zero; gases have non-zero pressures. Nevertheless, such pressures are negligible compared to their energy densities. The energy density is given by the mass density multiplied by c^2 from Einstein's mass-energy equivalence. As c is much larger than the speed of the ordinary matter, of which the pressure is proportional to the speed squared, the pressure is indeed negligible. Such matter, whose speed is much lower than the speed of light is called "non-relativistic matter" and its pressure can be neglected compared to the energy density. On the other hand, matter, whose speed is almost the speed of light is called "ultra-relativistic matter." Neutrino is an example. Notice that $\rho V^{4/3}$ must be (approximately) constant not only for photons but also for ultra-relativistic matter, because it must (approximately) satisfy $P = \rho/3$, which is derived from $E = pc$ which is (approximately) satisfied for ultra-relativistic matter.

I also want to remark that the ordinary matter must be treated as ultra-relativistic matter in the very early universe when the temperature of the universe was high enough for them to have kinetic energy that is much larger than its rest energy.

Problem 1. Let's denote the current matter density by ρ_{m0}, the current radiation density by ρ_{r0} and current scale factor by a_0. The current matter density is known to be about 3600 times larger than the current radiation density (i.e. $\rho_{m0} \approx 3600\rho_{r0}$). Then, what was the approximate scale factor when the two densities were equal?

Summary

- As our universe expands, the matter density decreases as $1/a^3$, and the radiation density decreases as $1/a^4$, where a is the scale factor.