Earth's gravity near its surface

In our earlier article, I briefly mentioned that the gravitational force around the Earth's surface is approximately given as follows:

$$F_g = -mg\hat{k} \tag{1}$$

Certainly, this can't be true for a place far from Earth's surface since the magnitude of the Earth's gravity should be much smaller than at the Earth's surface, whereas the above formula says that it is constant. Now, what would be the potential energy due to it? We naturally have:

$$\begin{aligned} U &= -\int F(\vec{r}) \cdot d\vec{r} \quad (2) \\ &= -\int (0, 0, -mg) \cdot (dx, dy, dz) \quad (3) \\ &= \int mgdz = mgz \quad (4) \end{aligned}$$

As z is the height, this formula agrees with our earlier formula for the gravitational potential energy in our earlier article "Potential energy and conservation of energy," which we derived rather cumbersomely.

Summary

- The gravitational force around the Earth's surface is approximately given by

$$F_g = -mg\hat{k}$$

- The corresponding gravitational energy is given by

$$U = mgz$$