Finding eigenvalues and eigenvectors

As advertised in "Eigenvalues and eigenvectors," we will briefly mention how one can obtain eigenvalues and eigenvectors of $n \times n$ matrix for any finite n.

As in the case of 2×2 matrices, eigenvalues and eigenvectors of $n \times n$ matrix A are defined as follows:

$$Ax = \lambda x \tag{1}$$

where λ is an eigenvalue, therefore a number. We can express the above equation as follows

$$(A - \lambda I)x = 0 \tag{2}$$

where I is an identity matrix. Now, we can use the property of determinant. The above equation has a non-zero solution x if and only if the following condition is satisfied:

$$\det(A - \lambda I) = 0 \tag{3}$$

For $n \times n$ matrix, this is nth order polynomial in λ. Therefore, in general, we will have n solutions for λ, the eigenvalues. After obtaining the eigenvalues, we can obtain corresponding eigenvectors by plugging the eigenvalues into (2) and solving it.

Let me conclude this article by mentioning that this method is not quite useful when the size of matrix is infinite which is often the case in quantum mechanics.

Problem 1. What is the eigenvalues and the eigenvectors of the following matrix?

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{4}$$

Summary

- Eigenvalues can be found by solving $\det(A - \lambda I) = 0$.