Electric charge and Coulomb force

If you run a comb through your hair on a dry winter day, remove it, and move it closer to your hair again, you may observe that the comb attracts your hair. Phenomena such as this are due to “electric charge” and were known to us as early as around 600 B.C., when the Greek philosopher Thales rubbed amber with woolen clothes and found out that the amber attracted small objects such as straw.

So, what is electric charge? There are two kinds of electric charge: positive and negative. If an object carries more positive charge than negative charge, then it is positively charged. If an object carries more negative charge than positive charge, it is negatively charged. If an object carries equal positive and negative charge, it is said to be “neutral.” In other words, the “net” charge is the total positive charge minus the total negative charge.

For example, an atom is usually neutral, as it has a positively charged center, called a nucleus, around which negatively charged electrons orbit, and the magnitude of these charges are equal. However, if an atom gains extra electrons, it becomes negatively charged, and if it loses electrons, it becomes positively charged. In reality, the net charge of any charged objects in our daily lives is very small compared to the total amounts of positive charge and negative charge carried in the atoms in the objects.

It is also known that if the signs of charge of two objects are the same (i.e. both positive or both negative,) they repel each other, while a positively charged object attracts a negatively charged object and vice versa. In our example, the comb gained extra electrons and the hair lost electrons, so they became negatively charged and positively charged respectively, which made them attract each other.

Also, from experiments by the French physicist Coulomb in the 18th century, it is known that the force of attraction or repulsion is inversely proportional to the distance between the objects. For example, if the distance is halved, the force is quadrupled, and if the distance is tripled, the force becomes one-ninth. Of course, if the charges are bigger, they attract or repel more. Think of it this way: Let’s say that an object A has charge q_1 and and object B has charge q_2. They are separated by a certain distance d, and the force between them is F. If you place an additional charge q_1 on object A, this additional charge will receive the additional force F, as the force between charge q_1 and q_2 separated by the distance d is F. Therefore, A has now an electric charge $2q_1$ and receives the Coulomb force $2F$. Suppose you add an additional charge q_1 once again. You will get an additional force F, (as you would every time you add charge q_1 on A.) Now the Coulomb force will be $3F$ and the charge of A will be $3q_1$. Therefore, you see that the Coulomb force on A is proportional to the charge of A. By similarly manipulating object B (adding additional charges q_2s on B), we

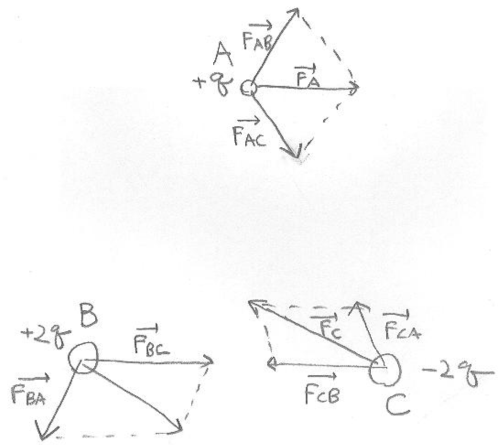

Figure 1: Coulomb forces among A, B, C.

can easily see that the Coulomb force on B is proportional to the charge of B. Therefore, we can conclude that the Coulomb force is proportional to the charge of A multiplied by the charge of B. Summarizing, we can express the magnitude of the "Coulomb" force as follows:

$$|\vec{F}_{12}| = |\frac{kq_1q_2}{r^2}| \tag{1}$$

where q_1 and q_2 are the electric charges of two objects respectively, r is the distance between them, F_{12} is the force exerted on object 1 by the presence of object 2, and k is a proportionality constant called "Coulomb's constant." The direction of the force is toward object 2, when q_1q_2 is positive (i.e. same sign, and attraction) while it is in the opposite direction when q_1q_2 is negative (i.e. different sign, and repulsion). Notice that the Coulomb force given by the above formula implies $|\vec{F}_{21}| = |\vec{F}_{12}|$ since

$$|\vec{F}_{21}| = |\frac{kq_2q_1}{r^2}| = |\frac{kq_1q_2}{r^2}| = |\vec{F}_{12}| \tag{2}$$

In other words, if object 1 attracts object 2 with, say 3N, (N is a unit for force and read "Newton") object 2 attracts object 1 with 3N as well. Similarly, if object 1 repels object 2 with, say 4N, object 2 repels object 1 with 4N as well.

In the presence of multiple charges, the resulting force is given by vector addition of each Coulomb force. (Remember? A force is a vector) See Fig. 1. You see that there are three objects: A, B, and C, whose electric charges are given by $+q$, $+2q$, and $-2q$ respectively. They are equal distances apart from each other. They attract and repel each other by the amount of forces dictated by (1). Here, as in (1), the notation $\vec{F}_{ij}$ denotes the Coulomb force exerted on object i by object j. Also, $\vec{F}_i$ is the total force exerted on object i. This is obtained by summing all the Coulomb forces exerted on object i by the other objects. For example, as in the figure, you see the following:

$$\vec{F}_A = \vec{F}_{AB} + \vec{F}_{AC} \tag{3}$$

$$\vec{F}_B = \vec{F}_{BA} + \vec{F}_{BC} \tag{4}$$

$$\vec{F}_C = \vec{F}_{CA} + \vec{F}_{CB} \tag{5}$$

You also see that $\vec{F}_{AB}$ is directed against B, since B and A both have a positive charge, so they repel each other. On the other hand, $\vec{F}_{AC}$ is directed toward C since C and A have opposite signs of electric charge, so they attract each other. Also, $\vec{F}_{BA}$ has the same magnitude as $\vec{F}_{AB}$, as explained before. Notice that the magnitude of $\vec{F}_{CB}$ is twice the one of $\vec{F}_{CA}$ as the charge of B is twice the one of A while the distance between C and A and the distance between C and B are same.

Problem 1. Let's say that a charge q is placed at $(0, 0, 0)$ and another charge $4q$ is placed at $(5d, 0, 0)$. Given this, exactly where does an object with charge Q need to be placed so that the electric force on this object is 0? (Hint: it needs to be placed on the x-axis.)

Problem 2. Let's say that there are only three charged objects A, B, C, and denote the total Coulomb force exerted on A by F_A, on B by F_B and so on, as before. Then, (3), (4), (5) are satisfied. Explain why $\vec{F}_A + \vec{F}_B + \vec{F}_C = 0$ is always satisfied no matter where A, B, and C are placed, and no matter the amount of charge A, B, and C carry. (Hint[1])

Generally, if there are only N charged objects 1, 2, 3 $\cdots, N-1, N$. Then $\vec{F}_1 + \vec{F}_2 + \cdots \vec{F}_{N-1} + \vec{F}_N = 0$ is always satisfied. Actually, this phenomenon is not restricted to the case of Coulomb force. The answer to Problem 2 will be provided in our later article "Newton's third law and the conservation of momentum with calculus."

Summary

- There are two kinds of electric charge: positive and negative
- Electric charges exert force one another. This is known as Coulomb force.
- If the signs of charge of two objects are the same, they repel each other. If the signs of charge of two objects are different, they attract each other.
- The magnitude of the Coulomb force between two objects is given by

$$|\vec{F}_{12}| = |\frac{kq_1q_2}{r^2}|$$

  where q_1 and q_2 are the electric charges of two objects respectively, r is the distance between them. The direction is along the line connecting two objects.

[1] Use (3), (4), (5) and $\vec{F}_{ij} = -\vec{F}_{ji}$.