Electric field

In our earlier article "electric charge and Coulomb force," we learned how much total force, say F_0, is exerted on an electric charge, say q_0, in the presence of n other electric charges, say $q_1, q_2 \cdots q_n$. Notice that the total force is proportional to q_0, as each Coulomb force due to each electric charge, say q_i is proportional to $q_0 q_i$, and the total force is the vector sum of each Coulomb force. This suggests us to define electric field $\vec{E}$ as follows:

$$\vec{E} = \frac{\vec{F_0}}{q_0} \tag{1}$$

which is equivalent to:

$$\vec{F_0} = q_0 \vec{E} \tag{2}$$

Here, notice that $\vec{E}$ is a vector, since a vector (i.e. $\vec{F_0}$) multiplied by a scalar (i.e. $1/q_0$) is a vector. Furthermore, we can easily check that $\vec{E}$ is independent of the value of q_0 since F_0 is proportional to q_0. For example, in the presence of single charge q_1, the electric field $\vec{E}$ at the point r meters away from it is given by:

$$|\vec{E}| = \frac{|\vec{F_0}|}{|q_0|} = \frac{|kq_0 q_1/r^2|}{|q_0|} = |\frac{kq_1}{r^2}| \tag{3}$$

This is the magnitude of the electric field. What is its direction? Observe following. Let's say q_0 is positive. Then, the direction of $\vec{F_0}$ is away from the charge q_1 when q_1 is positive and toward the charge q_1 when q_1 is negative. By applying this observation into (1), the direction of the electric field $\vec{E}$ is away from the charge q_1 when q_1 is positive and toward the charge q_1 when it is negative. The same can be similarly and easily concluded when q_0 is negative. In this case, the direction of $\vec{F_0}$ and $\vec{E}$ is opposite because q_0 is negative.

Now, let's consider the case that there are multiple number of charges. In the earlier article, we have seen that the total force was given by the sum of force due to each object. In other words,

$$\vec{F_0} = \vec{F}_{01} + \vec{F}_{02} + \cdots \vec{F}_{0n} \tag{4}$$

Dividing both-hand sides by q, we get:

$$\vec{E} = \vec{F_0}/q_0 = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots \frac{\vec{F}_{0n}}{q_0} \tag{5}$$

Figure 1: Three charges: A, B, C

Now let's define $\vec{E}_i = \vec{F}_{0i}/q_0$. In other words, $\vec{E}_i$ is the electric field in case only q_i were present. Then, we can re-express the above formula as:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots \vec{E}_n \tag{6}$$

In other words, the total electric field is given by the sum of the electric field due to each of the charge. Notice also that the magnitude of $\vec{E}_i$ is given as follows:

$$|\vec{E}_i| = |\frac{kq_i}{r^2}| \tag{7}$$

which is independent of q_0.

Let me give you some examples, because it sounds somewhat abstract without them. See Fig.1. We have three objects: A, B, and C, charged with $+q$, $-3q$, and $+2q$. What is $\vec{F}_B$, the total electric force exerted on the object B? Using Coulomb's law we see that it is given by:

$$\vec{F}_B = \frac{k \cdot 3q \cdot 2q}{r^2}\hat{x} - \frac{k \cdot 3q \cdot q}{r^2}\hat{x} \tag{8}$$

where $\hat{x}$ denotes the rightward direction.

However, we can reach at the same conclusion using the electric field picture. Using (7) and considering our earlier discussion on the direction of electric field, we can easily see that the electric field at the point B is given by

$$\vec{E}_B = \frac{kq}{r^2}\hat{x} - \frac{k \cdot 2q}{r^2}\hat{x} \tag{9}$$

Now, using (2), we conclude:

$$\vec{F}_B = -3q\vec{E}_B \tag{10}$$

which coincides with (8).

At first glance, it may seem cumbersome that we took two steps to calculate the Coulomb force by introducing the concept of electric field. Indeed, we could obtain the same result at a single step by using the definition of Coulomb force directly. However, later in the articles, we will see that the concept of electric field is powerful and necessary.

Finally, a vivid visualization. As we have shown the graphical picture of magnetic field in our earlier article "magnetic field," we can make the graphical picture of electric field in same way. See Fig.2. The direction of arrow is the direction of the force which a positive charge would feel, if it were present there. (i.e. the direction of the electric field) Notice that

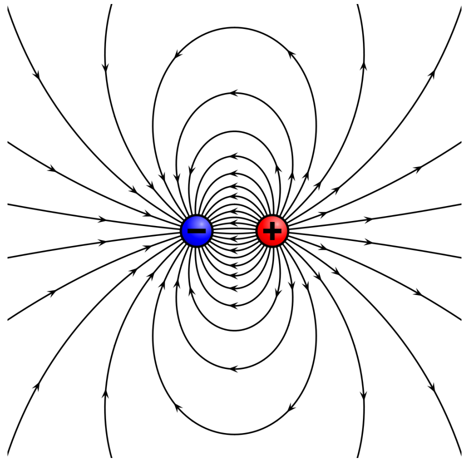

Figure 2: Graphical representation of electric field

the direction of the electric field is away from the positive charge and toward the negative charge, as explained earlier.

Problem 1. Two objects with each charge q and $4q$ are a distance d apart. Convince yourself that the electric field is zero at one point on the line joining them. How far is this point from each charge?

Summary

- If the total Coulomb force exerted on the point charge q is $\vec{F}$, the electric field at the point charge is given by

$$\vec{E} = \frac{\vec{F}}{q}$$

(The figure is from `http://commons.wikimedia.org/wiki/File:VFPt_dipole_electric_manylines.svg`)