Electric potential

Suppose a particle "A" with charge $q_A = q > 0$ is accelerated by a constant electric field $\vec{E} = E_x\hat{x}$ as in the figure. Supposing that the initial x coordinate of the particle is x_i and the final x coordinate of the particle is x_f, what is the gain of its kinetic energy during this trip?

Notice that this problem is very similar to the one we had in our earlier article "Potential energy and conservation of energy." The only difference is that now the force is in positive x-direction, whereas it was in negative z-direction there, and the magnitude of the force now is given by qE_x while it was mg there. Therefore, taking the similar step as before, we can easily see that the answer is given as follows:

$$\Delta K_A = qE_x(x_f - x_i) \tag{1}$$

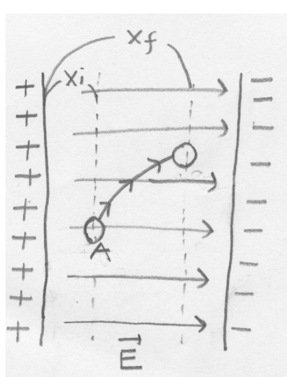
Now, suppose we have another particle "B" with charge $q_B = 2q$ and everything else is the same. What is the gain in the kinetic energy? It is easy. We just replace q by $2q$ in the above equation, and we get:

$$\Delta K_B = 2qE_x(x_f - x_i) \tag{2}$$

Now, suppose we have still another particle "C" with charge $q_C = -q$ and everything is the same. What is the change in kinetic energy? It is easy. By replacing q by $-q$ in (1), we get:

$$\Delta K_C = -qE_x(x_f - x_i) \tag{3}$$

So, actually, the kinetic energy decreases in this case. The first two cases would be similar to cases when a ball gains kinetic energy by falling down, while this case would be similar to



cases when a ball's kinetic energy decreases while soaring. Notice that the three equations can be re-expressed as follows:

$$\Delta K_\alpha = q_\alpha E_x(x_f - x_i) \tag{4}$$

where α can be A, B, or C. So, notice following. For the above equation, the part $E_x(x_f - x_i)$ is common, independent of the object. So, if you want to calculate how much the kinetic energy of any object with any charge will change, you do not need to calculate $E_x(x_f - x_i)$ every time. You can just calculate it for once, and simply plug in different values for q_is every time you calculate. Therefore, if we define electric potential as follows:

$$V = -E_x x \tag{5}$$

we have:

$$V_i = -E_x x_i, \quad V_f = -E_x x_f \tag{6}$$

which, then, we can write as follows

$$\begin{aligned} \Delta K &= qE_x(x_f - x_i) && (7) \\ K_f - K_i &= q(V_i - V_f) && (8) \\ K_i + qV_i &= K_f + qV_f && (9) \end{aligned}$$

Therefore, we can interpret the last equation as conservation of energy if we regard qV as electric potential energy. As an aside, the unit for electric potential is "Volt" which is a household name.

Problem 1. An object with charge $2C$ and mass 4kg is initially at rest at the position with electric potential 10 volt. If it receives electric force and lands at the position with electric potential 1 volt. What will be its final speed?

Summary

- In a constant electric field, given by $\vec{E} = E_x\hat{x}$, the electric potential is given by

  $$V = -E_x x$$

- The potential energy due to electric potential is given by

  $$qV$$

  where q is the electric charge.

- Then, the conservation of energy can be written as

  $$\frac{1}{2}mv_i^2 + qV_i = \frac{1}{2}mv_f^2 + qV_f$$