Electron magnetic moment

In the last article, we introduced the magnetic dipole moment. In this article, we will consider the magnetic dipole moment of electrons (also known as "electron magnetic moment"). In particular, we will try to connect it with the angular momentum of electron.

First, we will consider electron magnetic moment due to its orbital angular momentum. An orbital angular momentum of a particle is due to its actual motion. (There is what is called "spin angular momentum" which is not due to its actual motion as we will explain in later articles.) Here, we will treat it classically (i.e. without using quantum mechanics). In particular, we will assume circular orbit for electrons as in Bohr model. See Fig.1. Here, you see that the electron orbits counter-clockwise around the z axis with radius r and speed v.

Given this, let's calculate the magnetic moment. Remember our earlier formula $\mu = iA$. Remember also that the current i is given by the amount of electric charge passing a given cross section in circuit per unit time. As it takes $2\pi r/v$ seconds for the electron to rotate once, it is easy to see that electric charge $-e$, which is the electric charge of an electron, passes through a given cross section in circuit every $2\pi r/v$. Therefore,

$$i = \frac{-e}{2\pi r/v} \tag{1}$$

As the area is given by πr^2, we have:

$$\mu = iA = \frac{-e}{2\pi r/v} = -\frac{evr}{2} \tag{2}$$

Of course, it goes without saying that the direction of the magnetic moment here is along z axis. Using that the angular momentum in this case is given by $L = mvr$, we can express

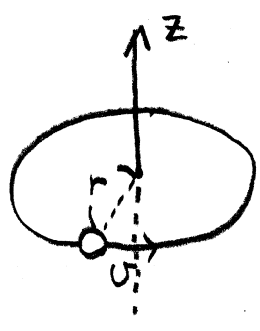

Figure 1: an electron on a circular orbit

the above equation as,

$$\mu = -\frac{e}{2m}L \tag{3}$$

Of course, it also goes without saying that the direction of the angular momentum here is along z axis. In particular, as the electron rotates counterclockwise the angular momentum is directed upward. On the other hand, the magnetic dipole moment is downward, as the current circulates clockwise. (Here, the direction of the current is direct opposite of the one in which electron moves, as the electron has a negative charge.) Therefore, considering these two directions, the above equation can be re-written using the vector notation as

$$\vec{\mu} = -\frac{e}{2m}\vec{L} \tag{4}$$

This formula holds well for the orbital angular momentum of electrons. However, it fails for what is called the "spin angular momentum." In particular, if we let the spin angular momentum by $\vec{S}$, the spin magnetic moment $\vec{\mu}_s$ is given by,

$$\vec{\mu}_s = -g\frac{e}{2m}\vec{S} \tag{5}$$

where g (called "g-factor") is given by

$$\begin{aligned} g_{\mathrm{exp}} &= 2.002319304362 \pm 0.000000000001 \\ g_{\mathrm{theory}} &= 2.002319304363 \pm 0.000000000002 \end{aligned}$$

where g_{exp} denotes the experimental value while g_{theory} denotes the theoretical value.

Two comments. First, that this value differs from 1 suggests that the model we have used to derive (4) is wrong for spin angular momentum; we can conclude that the spin angular momentum of electron is not due to physical rotation of electron.

Second, Dirac equation, which is the relativistic version of Schrödinger's equation for spin 1/2 particle, predicts $g = 2$ as we will see in our later article "Electron magnetic moment from the Dirac equation." However, as you see, g is not exactly 2. Physicists calculate g by Taylor-expanding it in terms of fine structure constant α. In other words,

$$g = 2 + c_1\alpha + c_2\alpha^2 + c_3\alpha^3 + \cdots \tag{6}$$

where physicists calculate cs from a theory called "QED(quantum electrodynamics)." c_1, which is $1/\pi$, was first obtained by Julian Schwinger in 1948. This result is engraved on his tombstone. However, it is hard to calculate higher-order terms by using Schwinger's method. Nowadays, physicists use Feynman's method. Also, as you can see, the theoretical value and the experimental value for g-factor agree very well. It is a triumph of theoretical particle physics.

Summary

- A model based on classical physics can predict the ratio between the magnetic moment of a particle and its angular momentum. However, in case of the spin magnetic moment of electron, and the spin angular momentum of electron, the actual ratio differs about by a factor 2 from the classical prediction. This factor is called g-factor, and quantum electrodynamics can predict this value very precisely, which agrees with the experiment to a very high precision.