What is entropy? From a macroscopic point of view

If you put a cold object in contact with a hot object or mix them, the cold object will gain energy (i.e. heat) from the hot object. In other words, the hot object will lose energy to the cold object.

Indeed, you would be surprised if the cold object lost energy to the hot object and got colder and colder while the hot object gained energy and got hotter and hotter. The second law of thermodynamics says that this scenario is indeed impossible. In 1854, Clausius stated this law as follows: "Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time." In other words, heat cannot spontaneously flow from a cold object to a hot one. You may say that heat flows from a cold to a hot object in the case of a refrigerator, but this doesn't count as "spontaneous" since you have to use extra energy (i.e. electricity) to achieve this. There is another equivalent statement for the second law of thermodynamics: "The total entropy always increases." To understand this statement, we have to understand what entropy is. In an earlier article, I explained what entropy is from a microscopic point of view, from the standpoint of molecules. In this article, I will forget about molecules and talk about what entropy is from a macroscopic point of view, i.e. in our daily lives.

In a macroscopic setting, entropy is defined as follows:

$$\Delta S = \Delta Q/T \tag{1}$$

Here S is entropy, and ΔS means the change in entropy. ΔQ is the flow of heat. If an object gains heat, ΔQ is positive. If the object loses heat, ΔQ is negative instead. T is the temperature of the object measured in Kelvin. The kelvin is the unit of temperature in the Kelvin scale, which is a scale for temperature like the Celsius and Fahrenheit scales. The difference between the temperature measured in kelvins and in Celsius or Fahrenheit degrees is that while the latter two can have negative values, such as -10 degrees Celsius or -10 degrees Fahrenheit (written -10° C and -10° F respectively), temperature measured in kelvin can "almost never" have negative values. This is because zero kelvin, written 0 K, is set at the temperature known as absolute zero; this is the coldest possible temperature. (Negative temperature in the Kelvin scale, i.e. a temperature below absolute zero, is very rare. However, in a later article "The definition of temperature," you will see that a system with a negative temperature is actually *hotter* than any system with a positive temperature.) -273° C is 0 K and 0° C is 273 K. So if you add 273 to the Celsius degrees you get the temperature in kelvins. In other words, one can never have a system colder 273°C.

To help you concretely understand the above formula, let me give an example. Let's say that a cold object whose temperature is $-10°$C, (i.e. 263 K) and a hot object whose temperature is 30°C (i.e. 303 K) are in contact with each other. If 100 joules of energy is transferred from the hot object to the cold object, ΔQ for the cold object will be 100 J and ΔQ for the hot object will be -100 J. Therefore, the total change of entropy will be

$$\Delta S = \frac{100}{263} + \frac{-100}{303} = 100\left(\frac{1}{263} - \frac{1}{303}\right) \tag{2}$$

which is bigger than zero. So, we indeed see that the total change of entropy is positive in this case. Now, I want to comment on several things:

1) If 100 J of heat spontaneously flowed from the cold object to the hot object, the total change of entropy would be negative as

$$\Delta S = -\frac{100}{263} + \frac{100}{303} = 100\left(\frac{1}{263} + \frac{1}{303}\right) \tag{3}$$

As already stated, this doesn't happen as entropy would decrease in this case.

2) If we used Celsius or Fahrenheit instead of Kelvin for the entropy formula, it wouldn't make any sense because we would have negative temperatures. In our example, it would be

$$\Delta S = \frac{100}{-1} - \frac{100}{30} \tag{4}$$

which is less than zero. This justifies the use of Kelvin instead of Celsius or Fahrenheit.

3) In the case of a refrigerator or an air conditioner, heat flows from a cold object to a hot object, but the heat lost by the cold object must be much less than the heat gained by the hot object to satisfy the condition that entropy always increases. For example, if a refrigerator whose temperature was 5 °C (i.e. 268 K) lost 100 J of heat, the surrounding system or the vicinity whose temperature was 25 ° C (i.e. 298 K) must have gained heat of at least 100 J $\times$ 298/268 = 111 J. For example, if the vicinity gained 105 J, which is less than 111 J, this wouldn't happen as

$$\Delta S = -\frac{100}{268} + \frac{105}{298} \tag{5}$$

would be less than zero. I also want to remark that the difference between the heat gained and the heat lost comes from the energy you use to cool down the refrigerator (i.e. electricity). For example, if the refrigerator lost 100 J and the vicinity gained 150 J, you must have used 50 J of electricity to cool down the refrigerator. This follows from the conservation of energy. See Fig. 1. In other words, given the fact that in our preceding example the vicinity must have gained at least 111 J of heat, we must have used at least 11 J (= 111 J $-$ 100 J) of energy to pump out the 100 J from the refrigerator; you need energy to make a refrigerator work. There is no such thing as a "free lunch."

4) The second law of entropy can be applied to the heat engine. A heat engine absorbs heat from a hot object and transforms this energy into mechanical work. An example is the car engine, which absorbs energy from hot gasoline. However, in this case as well, the second law of entropy says that there is no "free lunch." To illustrate this point, let's say that a

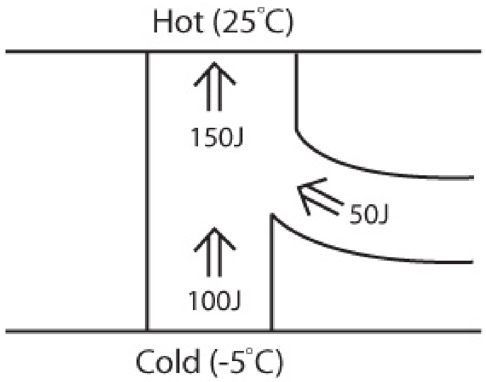

Figure 1: air condition

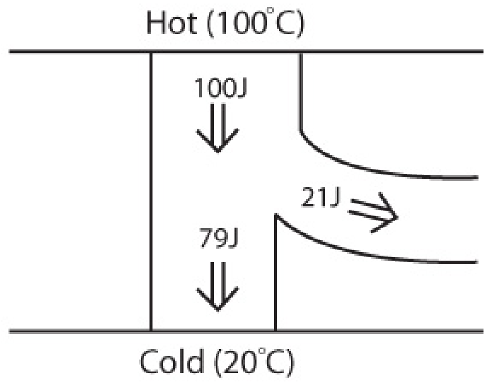

Figure 2: heat engine

hot object is 100 °C (373 K) and the vicinity is 20 °C (293 K). If the engine absorbed 100 J from the gasoline, transformed this heat to 100 J of the mechanical energy and dispensed no energy to the vicinity, it would be violating the second law of thermodynamics since the total change of entropy would be negative, as follows.

$$\Delta S = -\frac{100}{373} \tag{6}$$

(Remember that the hot object lost 100 J, so the sign is negative.) To make this work, the vicinity must absorb some energy to make the total change in entropy positive. See Fig. 2. In our example, at least 100 J × 293/373 = 79 J must be absorbed by the vicinity. For example:

$$\Delta S = \frac{79}{293} - \frac{100}{373} \geq 0 \tag{7}$$

The difference between the energy you absorbed from the hot object and the energy you dispensed to the cold object is the mechanical work you can use. Again, this follows from the conservation of energy. In this case as well, there is no free lunch, as there is no way to change all the heat to mechanical work. If you absorb 100 J from the hot object, you can use at most 21 J (= 100 J − 79 J) of mechanical work.

5) Our definition of entropy in a macroscopic setting ($\Delta S = \Delta Q/T$) coincides with our earlier definition of entropy in a microscopic setting ($S = k \ln W$). However, this correspondence was only achieved by the combined work of many physicists in the 19th century.

Problem 1. Suppose a heat engine that absorbed Q_H joules from a hot object with temperature T_H Kelvin, and dispensed certain energy to the vicinity with temperature T_C Kelvin. Find the maximum mechanical work this heat energy can transform from the initial Q_H joules.

6) A perpetual motion machine is a hypothetical machine that can do work perpetually without an energy source. However, we know that such a machine doesn't exist as it would violate either the first law of thermodynamics or the second law of thermodynamics.

You might want to say that a waterwheel is an example of a perpetual motion machine but it is not. A waterwheel is a wheel that constantly moves because water is constantly pouring down above the waterwheel. See the figure below.

However, it is not moving without an energy source. There needs to be water constantly pouring down, and that water, which has a potential energy, is the energy source. In the earlier article on the conservation of energy, I mentioned that a ball at a high place has a potential energy because such a potential energy can be converted into kinetic energy when the ball is dropped. Similarly, the water above the waterwheel has a potential energy, and the potential energy of the water is converted into the kinetic energy of the waterwheel. Moreover, to make it perpetually move, either water needs to be constantly poured down, in which case the waterwheel will stop moving if water is running out, or somebody needs to constantly raise the fallen water above the waterwheel, which requires energy.

Many people claimed that they had invented a perpetual motion machine, but they all turned out to be frauds. For example, they used hidden batteries or made other persons move them behind the scene.

In the United States, a patent applicant is not usually required to submit a model to demonstrate its operability, but there is an exception for cases involving perpetual motion. In France, patent applications for a perpetual motion machine is not permissible since 1775.

Summary

- $\Delta S = \dfrac{\Delta Q}{T}$. ΔS is the change in entropy, ΔQ is the flow of heat. (If an object gains heat, ΔQ is positive.) T is the temperature.
- That entropy always increases implies that spontaneously heat always flows from the cold object to hot object.
- To make heat flow from a cold object to a hot object, we need to do extra work. Otherwise, it would violate the condition that the entropy always increases.
- We cannot convert all the heat to mechanical work, but only part of them. Otherwise, it would violate the condition that the entropy always increases.

- A perpetual motion machine is a machine that can do work perpetually without an energy source. Such a machine doesn't exist as it would violate either the first law of thermodynamics or the second law of thermodynamics.