Equilibrium of forces

See the seesaw in Fig. 1. An object with weight 30 N (i.e. if the mass of the object is m, the weight is given by $mg = 30$ N) is placed 10 meters left of the pivot point, and another object with unknown weight is placed 15 meters right of the pivot point. Assuming the seesaw is in equilibrium (i.e. not moving), what is W the weight of the object placed on the right? And what is F the force the pivot exerts on the rod? Let's solve this problem. We know that the total torque must be zero. Since the object on the left gives the torque 30 N$\times$ 10 m=300 N·m anti-clockwise, the object on the right should give the torque 300 N·m clockwise to balance the seesaw. Thus, we get

$$300 \text{ N} \cdot \text{m} = 15 \text{ m} \times W \tag{1}$$

Thus, we get $W = 20$ N. What is F? Since the total force exerted on the seesaw must be zero, we get

$$F - 30 \text{ N} - W = 0 \tag{2}$$

Thus, $F = 50$ N.

There are other ways to solve this problem. We just regarded the pivot point as the rotation of axis. However, as in the last article, it turns out that we can regard another point as the rotation of axis, and still be able to solve the problem. To this end, recall our earlier formula, which we reproduce here for convenience.

$$\frac{dL'}{dt} = \sum_i \vec{r}_i' \times \vec{F}_{i(\text{ext})} - \left(\sum_i m_i \vec{r}_i' \right) \dot{\vec{v}}_0 \tag{3}$$

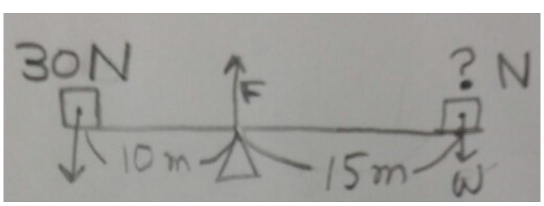

Figure 1: A balanced seesaw

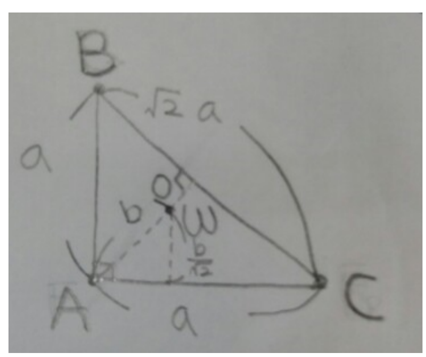

Figure 2: A balanced right isosceles triangle-shaped table

Now, notice that we are solving an equilibrium problem. Thus, $\dot{\vec{v}}_0$ in the above formula, which denotes the acceleration of the rotation of axis is zero, as nothing is moving. Moreover, as nothing is moving, the left-hand side of the above equation is also zero. Thus, we conclude

$$0 = \sum_i \vec{r}_i' \times \vec{F}_{i(\mathrm{ext})} \tag{4}$$

Given this, let's solve this problem by regarding the point where the object with weight 30 N is located as the axis of rotation. As the total torque must be zero, we have

$$10 \text{ m} \times F - (10 \text{ m} + 15 \text{ m}) \times W = 0 \tag{5}$$

Also, as the total force must be 0, we have the condition (2). So, we have two unknowns and two linear equations. We know how to solve them. You will get $F = 50$ N and $W = 20$ N.

Problem 1. Solve this problem again by regarding as the rotation of axis the point where the object with weight W is located.

Now, let's solve another problem. See Fig. 2. You see a right isosceles triangle-shaped table. The sides are a, a, $\sqrt{2}a$ meters and the legs are located at the vertices A, B, C. If you place an object with weight W at the place marked with O, which is b meters away from the vetex A, and $b/\sqrt{2}$ meters away from $\overline{AC}$, what are F_A, F_B, F_C the forces exerted on each leg? Assume that the mass of the table itself is negligible.

From the symmetry, we can easily see that $F_B = F_C$. Also, if you regard the axis of rotation as $\overline{AC}$, then the total torque is given by

$$aF_B - \frac{b}{\sqrt{2}}W = 0 \tag{6}$$

As the total force must be zero, we have

$$F_A + F_B + F_C = W \tag{7}$$

Thus, we get

$$F_B = F_C = \frac{b}{\sqrt{2}a}W, \quad F_A = \left(1 - \frac{\sqrt{2}b}{a}\right)W \tag{8}$$

Problem 2. Solve the problem in "Physics solutions for mathematical problems." (Hint:[1])

Summary

- For an object to be in equilibrium, both the total force being exerted on it, and the total torque being exerted on it must be zero.

[1] Let s be the length of each side. Then, express F_A in terms of a, s, Mg, express F_B in terms of b, s, Mg, and express F_C in terms of c, s, Mg. Finally, use the condition $F_A + F_B + F_C = Mg$.