Equivalence principle

Suppose you are inside an elevator and suddenly the rope that suspends the elevator is cut off. What will happen to you inside? How will you feel? Suppose you drop an apple at this point. See Fig. 1. Then, the apple will fall downwards. However, the elevator is also freely falling downwards because the rope is cut off. Since the apple and the elevator will fall at the same rate, you, who are inside the elevator, will see that the apple is on the air and doesn't fall.

In other words, you will experience no gravity. Furthermore, if you step on a scale in the elevator, the scale will measure your weight to be zero, since the scale is free falling very fast along with the elevator, so it will be hard for you to step on it with a weight. In other words, you also, not only apple, will experience weightless state.

Einstein noted that you, who are inside the elevator, would not be able to know whether you are on space where no gravity exists or you are inside an elevator freely falling. Of course, assuming that there is no window, through which you can see the outside of the elevator. Indeed, you may be familiar with the fact that astronauts inside spacecraft rotating around the Earth experiences weightlessness. This is so, not because the gravity is zero in the space, but because the spacecraft is "freely falling." (As the spacecraft is too fast, "freely falling" becomes the rotation around the Earth. The orbit of the spacecraft is curved due to gravity, but the Earth is curved, so it stays on the same height.) Similarly, to train astronauts comfortable with weightless state, they are taken on an airplane, and then make airplane

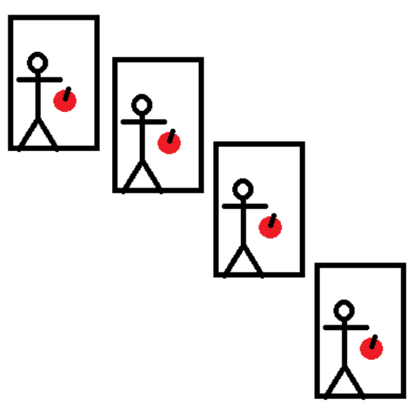

Figure 1: If you are in a freely falling elevator, you do not feel any gravity, as you are falling together with the elevator; if you drop an apple it will just stay in the middle of the elevator.

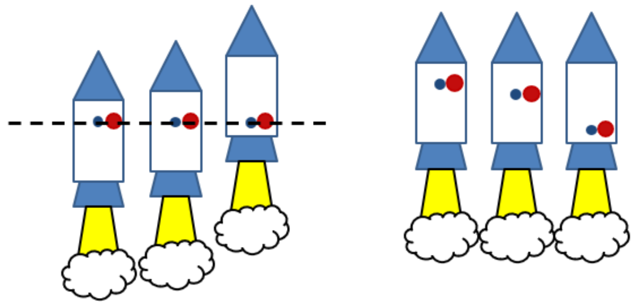

Figure 2: On the left, we see the view of an observer outside the spacecraft. Objects are just staying at the same positions. Only the spacecraft is moving. On the right, we see the view of an observer inside the spacecraft. Instead of the spacecraft, the objects seem to be moving, actually, falling.

freely fall by shutting off the engine.

Now, let's say that a spacecraft (or an elevator) is in a space where no gravity exists, but the spacecraft is accelerating upwards. See Fig. 2. Then, an observer inside the spacecraft will feel as if the gravity is exerting force downwards. Let me explain the reason why. Suppose there is an apple, initially at rest, inside the spacecraft. To the observer outside the spacecraft, the apple stays there because there is no gravity. See the left picture of Fig. 2. However, as the spacecraft is accelerating upwards, the observer inside the spacecraft will see that the apple is falling downwards. This is easy to understand as the floor of the spacecraft approaches the apple faster and faster and eventually the floor will hit the apple. At this point, the observer inside the spacecraft will think that the apple has dropped to the floor. Again, the observer inside will not know whether it is gravity or the acceleration of elevator that makes the apple falling downwards, as long as there is no window in the spacecraft.

Notice also that this implies that all the objects in the spacecraft experience the same acceleration, because they stay there and the spacecraft is the only one that moves upward. (In Fig. 2, we drew another object on the left side of the apple.) Thus, relative to the spacecraft moving upwards, everything is meant to experience the same acceleration.

This is important because this also happens to be the property that the Newtonian gravity actually follows. Let me explain why. Remember that, according to Newton, the gravitational force acting on an object with mass m is given by

$$\vec{F} = \sum_n \frac{G m_n m}{r_n^2} \hat{r}_n \tag{1}$$

where m_n is the mass of the nth object, r_n is the distance to the nth object, and $\hat{r}_n$ is a unit vector (i.e., magnitude 1) with the direction to the nth object. Then, as we have

$$\vec{F} = m\vec{a} \tag{2}$$

the acceleration is given by

$$\vec{a} = \sum_i \frac{Gm_i}{r_i^2}\hat{r}_i \tag{3}$$

Here, we see that m is "canceled out." Thus, according to Newton, the gravitational acceleration of an object doesn't depend on its own mass. This is due to the fact that the gravitational force (1) is proportional to m.

Anyhow, this idea that the gravitational acceleration doesn't depend on the mass of the object being tested, but only on its location led to Einstein to regard the gravitation as an acceleration of frame (i.e. such as elevator or spacecraft). This idea was crucial to the development of general relativity by Einstein.

Also, this idea means the complete equivalence between gravitational mass and inertial mass. Let me explain what they are. Simply speaking, m in (1) is the gravitational mass and m in (2) is the inertial mass. Let me re-write them here.

$$\vec{F} = \sum_n \frac{Gm_n m_g}{r_n^2}\hat{r}_n = m_i\vec{a} \tag{4}$$

Thus, m_g and m_i are not canceled, and the gravitational acceleration $\vec{a}$ depends on the ratio of m_g to m_i as follows

$$\vec{a} = \frac{m_g}{m_i}\left(\sum_n \frac{Gm_n}{r_n^2}\hat{r}_n\right) \tag{5}$$

If $\vec{a}$ doesn't differ from objects to objects, it means the complete equivalence (or equality) between the gravitational mass and the inertial mass as

$$\frac{m_g}{m_i} = 1 \tag{6}$$

As mentioned earlier, we know that gravitational acceleration is about $9.8\mathrm{m/s^2}$, regardless of the objects tested. It indeed means that the gravitational mass and the inertial mass are the same. Or, are they? In the next next article, I explain what kind of experiments physicists performed to check the equivalence principle.

Summary

- If you are in an elevator freely falling, you will experience no gravity.
- Then, you would not be able to know whether you are on space where no gravity exists or you are inside an elevator freely falling.
- Now, let's say that an elevator is in a space where no gravity exists, but the elevator is accelerating upwards. Then, if you drop an apple there, you will not know whether it is gravity or the acceleration of elevator that makes the apple falling downwards.
- This implies, all the objects in the elevator experience same acceleration, because the elevator is the only one accelerating.

- That the objects experience same acceleration is also the property of gravity.
- This implies the inertial mass and the gravitational mass are the same.