Experiments to test equivalence principle

In our earlier article "Equivalence principle," I explained that the equivalence principle means that the gravitational mass m_g and the inertial mass m_i are the same. In other words, it is a principle that says that the gravitational acceleration of an object doesn't depend on the object's mass or material.

Thus, if we find that the gravitational accelerations of two objects located at (almost) the same position are different, we confirm the violation of the equivalence principle. Then, how can we quantify this violation of the equivalence principle? The bigger the difference between the two objects' gravitational accelerations, compared to their average gravitational acceleration, the more equivalence principle is violated. Therefore, we define

$$\eta(a,b) = \frac{g_a - g_b}{(g_a + g_b)/2} \tag{1}$$

where $g_a(g_b)$ is the gravitational acceleration of object $a(b)$. Here, you see that the numerator is the gravitational acceleration difference of the two objects, and the denominator is the average of their gravitational acceleration. Thus, the bigger η, the more severely the equivalence principle is violated. If the equivalence principle is not violated at all, we have $\eta = 0$.

Problem 1. Show that (1) can be re-expressed as

$$\eta(a,b) = 2\frac{(m_g/m_i)_a - (m_g/m_i)_b}{(m_g/m_i)_a + (m_g/m_i)_b} \tag{2}$$

where $(m_g/m_i)_a$ denotes the ratio of m_g/m_i for object a, and similarly for $(m_g/m_i)_b$.

So, how can we measure g to check the equivalence principle? The simplest way would be dropping two balls simultaneously, and see if they arrive at the ground at the same time. However, it is not a good way to check the equivalence principle, because it takes only several seconds for the balls to drop even from a high building. Unless η is big enough, the arrival time difference for the two balls will be too small to notice. Moreover, the air resistance will affect the falling time much more than the difference in g does.

Actually, there is a better method. In 1686, Isaac Newton tried to detect the violation of equivalence principle by pendulum. In a later article, you will see that the period of a pendulum with length l is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{3}$$

Thus, by measuring the period of a pendulum, one can measure the gravitational acceleration g.

By the way, a good way to accurately measure the period is not measuring how much time it takes to swing once, but measuring how much time it takes to swing many number of times, such as ten thousand or so on. Let me explain why. Let's say the error in measuring is about 0.1 second, and the period is about 1 second. If you measure the time to take swing just once, you obtain that the period T is (1.0 ± 0.1) second. However, if you measure the time for ten thousand swings, you will get something like $10000\ T = 10000 \pm 0.1$ seconds, which implies $T = 1.00000 \pm 0.00001$ seconds. Now, you have 10,000 times better accuracy.

Anyhow, Newton tried many different materials for the pendulum, but could not find the violation of equivalence principle. He concluded that η is smaller than 0.001. In 1832, Friedrich Bessel performed the same pendulum experiments with more accuracy and many more different materials. He concluded that η was smaller than 1 part in 50,000 (i.e., 2×10^{-4}).

This was the limit to the pendulum method. Scientists were not able to test the equivalence principle more accurately, if they held on to the same method. Lucikly, in 1885, the Hungarian physicst Loránd Eötvös made a breakthrough. He used the fact that the Earth is rotating, which implies that there are two forces acting on an object on the Earth: the gravitational force and the centrifugal force. See Fig. 1. As the gravitational force is proportional to m_g, while the centrifugal force is proportional to m_i due to its inertial force origin, we will be able to obtain the ratio m_g/m_i, if we measure the ratio between these two forces. If this ratio is different for different objects, then the equivalence principle is violated.

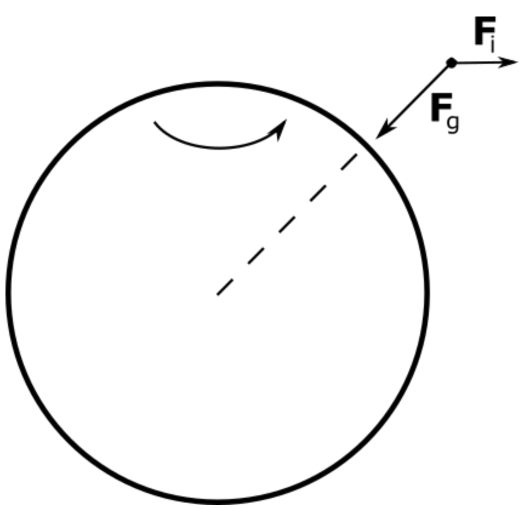

Figure 1: $\vec{F}_g$ is the gravitational attraction toward the center of the Earth. $\vec{F}_i$ is the centrifugal force due to the rotation of the Earth. Notice that $\vec{F}_i$ is perpendicular to the rotation axis of the Earth.

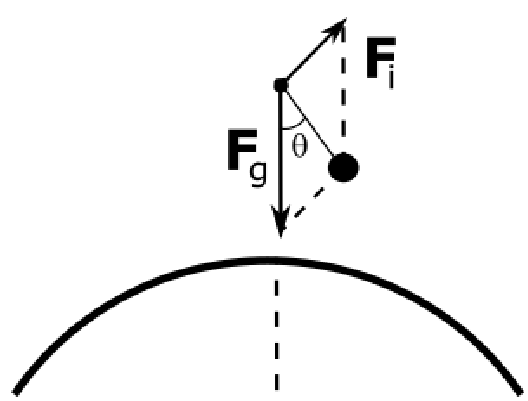

Figure 2: A close view of Fig. 1. θ is the angle between the direction toward the center of the Earth and the direction of the pendulum. In reality, $\vec{F}_i$ is much smaller than F_g, thus θ is quite small.

How can we know the ratio between these two forces? See Fig. 2. If you suspend a pendulum by a string, the angle between the direction toward the center of the earth and the direction in which the pendulum is suspended will be determined by the ratio of these two forces. Thus, if we carefully measure the direction to which the ball is suspended, we will be able to check the violation of the equivalence principle.

Problem 2. Assuming $m_g = m_i$, obtain θ in Fig. 2. at the latitude 45°. Use the fact that the radius of the Earth is about 6400 km, the rotation period of the Earth is about 24 hours, and the gravity on the Earth is about 9.8 m/s^2.

Problem 3. Now, let's consider the case m_g is not equal to m_i. For η, 1 part in 20 million, what is the difference between their θs?

Actually, this η was the accuracy Eötvös achieved. He concluded that η was smaller than this. If you calculate the answer accurately, with accurate values, for 47.5°, the latitude of Budapest, where Eötvös performed his experiment, you will obtain that the expected θ difference is about 1/60000 seconds (3600 seconds is 1°). Was he really able to measure such a small angle? In radians, it is about 8×10^{-11}. Thus, if the ball was suspended by a thread, say, 1 meter long, the violation of equivalence principle by this η would have caused the position difference of the ball of only 8×10^{-11} meter. The size of atom is about 10^{-10} meter, so the difference would be smaller than the size of atom. Considering that the atom hypothesis was only confirmed in the early 20th century, not to mention that there was no technology to take a picture of atom, Eötvös must have indeed devised a clever trick to achieve so high accuracy. What is it?

He used a torsion balance. See Fig. 3. Two equal masses are attached at the ends of the rod. If each force acting on each ball is not pointing in the same direction, the fiber receives a torque, which twists it. By measuring how much the fiber is twisted, one can figure out the torque.

This is the basic idea of the Eötvös method. Of course, there are some technical details. First of all, once you set the experiment apparatus, the fiber will be twisted by torque accordingly. But, how much? You do not know, because it is already twisted. To measure this angle, you should compare the state of fiber when the torque is applied with the one when the torque is not applied. Or alternatively, the one when the torque is applied with the one when the opposite torque is applied. Of course, in this alternative case, the angle difference will be twice the value of the original one.

What Eötvös did was smoothly rotating the rod to which the two balls are attached, by 180 degrees. So, he turned Fig. 3 into Fig. 4. Now, the opposite torque, but with the same magnitude is applied. Thus, he could measure how much the fiber is twisted this time, compared to the situation before the 180° rotation.

Nevertheless, the angle twisted would be still small. How did he measure it? He attached a mirror to the fiber, so that a light beam reflecting on the mirror rotated by a small angle could make a big position difference on the point the light beam reaches. See Fig. 5. He put a telescope to observe the light. His apparatus was sensitive up to 1 minute (60 minutes is

1°), but he didn't see any of such deflection. The torque due to the force direction difference of 1/60000 second would have twisted the fiber by 1 minute, which he didn't observe. That's how he concluded that η was smaller than 1 part in 20 million, or $\eta < 5 \times 10^{-8}$.

Thus, Eötvös didn't detect the violation of equivalence principle. Was he in vain? No. Not only did he invent the method to test the equivalence principle, which was employed by other scientists, but also his device was used, not to test the equivalence principle, but to check how rapidly gravity changes as you change your position. This change is called "gravitational gradient." This was important, because engineers can find the location of oil or certain minerals beneath the ground by carefully measuring gravitational gradient.

In 1964, Roll, Krotkov and Dicke at Princeton University, developed another novel method

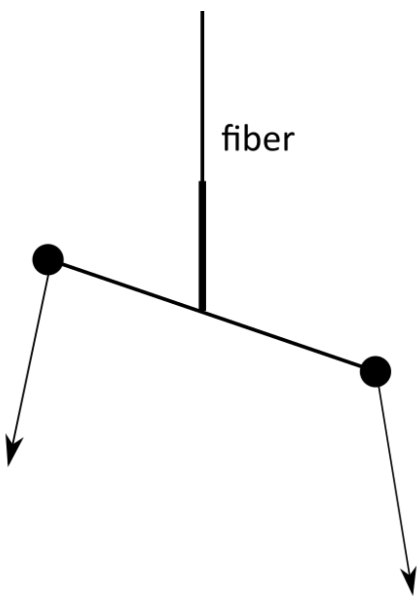

Figure 3: Two balls of equal mass are attached to the torsion balance. If the direction of the forces acting on these two balls are different, the fiber receives a torque.

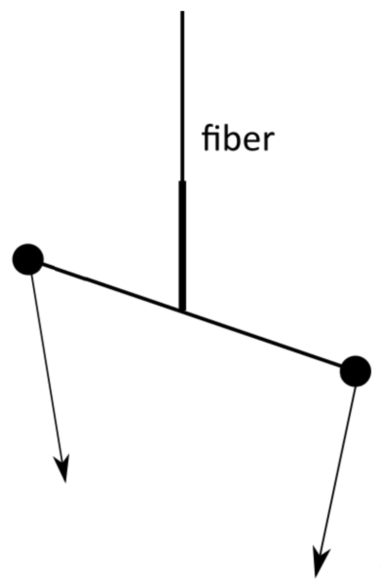

Figure 4: Same as Fig. 3, but the positions of the balls are exchanged.

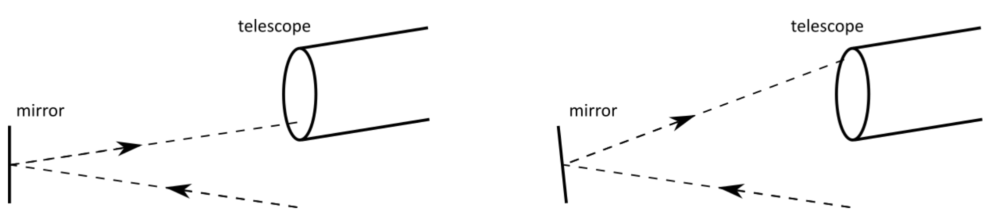

Figure 5: On the left, a light beam is reflected by an unrotated mirror. On the right, a light beam is reflected by a rotated mirror. The rotation of the mirror affects the final beam's position, which is checked by the telescope.

to check the equivalence principle. Remember that Eötvös had to rotate the rod with two balls by 180°. He had to make an apparatus for the rotation. This time, the Princeton physicists didn't need to make this complicated apparatus, because they found out a way to rotate the two balls "automatically" and "gently." The two balls are on the Earth, so relative to the Sun, they rotate as the Earth rotates; instead of measuring the gravity of the Earth, they concerned with the gravity of the Sun. The Sun "seems" to rotate around the apparatus once a day, so they used this fact to check whether the gravitational attraction due to *the Sun*, not due to the Earth, follows the equivalence principle. They had to consider and regulate many effects that could affect the experimental results, such as gravitational gradient, ground vibration, temperature and so on. Anyhow, they used electric devices to record the data and computer to analyze them. They obtained $\eta = (1.3 \pm 1.0) \times 10^{-11}$ for gold and aluminum.

In a later article, I will approach matter discussed here more quantitatively. We will explicitly calculate the torque in terms of η and so on.

Summary

- η, the parameter that quantifies the violation of the equivalence principle is given by

  $$\eta(a,b) = \frac{g_a - g_b}{(g_a + g_b)/2}$$

  where $g_a(g_b)$ is the gravitational acceleration of object $a(b)$.

- When η is 0, there is no violation of the equivalence principle.

- Eötvös used the centrifugal force due to the rotation of the Earth to perform experiments to check the equivalence principle.