History of Astronomy from the late 17th century to the early 20th century

Determination of the size of the Solar System

The exact size of our Solar System was not known until the late 17th century. True, Kepler used the size of orbits of the planets, when he deduced Kepler's third law. However, they were ratios of the size of orbits, not the actual sizes. Thus, once one can measure any one of the actual sizes of the orbits, the rest can be determined.

So, how can one measure these distances? Using parallax. The invention of telescopes made astronomical observation precise enough for it. In 1672, both Giovanni Cassini and John Flamsteed used different methods to measure the parallax of Mars during opposition. Remember that Mars is closest to the Earth during opposition, so it was a good chance. Cassini sent Jean Richer to the Cayenne island in South America a year before, to measure the island's latitude and longitude. Then, Cassini in Paris and Richer in the Cayenne Island simultaneously observed Mars during opposition. The parallax was 15 arcseconds.

If Cassini used the distance between two different points on the Earth as a source of the parallax, Flamsteed used the motion of the Earth as one. He observed Mars a little after sunset one evening, and then a little before sunrise the following morning. Of course, Mars is moving, so he chose the day when the apparent movement of Mars momentarily stops. You will know what I mean if you read "The Solar System and the apparent motion of planets." This is when Mars begins or ends retrogradation. Both Cassini and Flamsteed got quite good values. They were about 10% different from the correct one.

In 1663, Scottish mathematician James Gregory came up with the idea of using the transit of Venus or Mercury to determine the distance to the Sun by measuring the parallax between different points on the Earth. Transit is the phenomenon that Mercury or Venus passes across the Sun, which makes them visible in the background of the Sun. This allows precise measurement because the apparent position of Mercury and Venus can be compared with that of the Sun. In 1691, Halley established the laws that predicted the time of transits. In 1716, he suggested that scientists observe the transits of Venus in 1761 and 1769 to determine the distance to the Sun. They are rare events that happen twice every 243 years. In 1761 and 1769, long after Halley's death, scientists observed the transit all around the world. In 1771, the distance to the Sun was finally obtained, and it agrees with the modern value within 1%. We will talk more about Halley later in the article.

Sir Isaac Newton

In our earlier articles "A short introduction to the history of physics, and string theory as a "Theory of Everything"" and "History of astronomy up until the mid 17th century," we already mentioned that Newton explained Kepler's three laws, which is the beginning of physics. He published these findings in his Latin book *Philosophiæ Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy).* This book is often called *Principia.*

More specifically, what Newton mathematically showed is the following: the orbit of any planets around the Sun should be a conic section with one of the foci located at the Sun if the Sun's gravitational force on the planet is directed right toward the Sun and inversely proportional to the square of the distance between the planet and the Sun. (For example, if the distance between the Sun and the planet is doubled, the gravitational force is quartered.) So, this explains Kepler's first law. The same hypothesis explains Kepler's third law if one additionally assumes that the Sun's gravitational force on the planet is proportional to the mass of the planet. However, the easiest one to actually explain is Kepler's second law. Newton showed that a planet must follow Kepler's second law if the Sun's gravitational attraction on the planet is directed right toward the Sun; it doesn't matter how big or small the gravitational attraction is.

If you are interested in Newton's proof of Kepler's first law, you might want to read the English translation of Newton's book *Principia* originally written in Latin. However, if you have difficulty understanding it, like the late Nobel laureate in physics Richard Feynman, you may want to read his book *Feynman's Lost Lecture: The Motion of Planets around the Sun*, in which he followed *Principia* as far as he could and created his own proof for the part he couldn't understand.

These are geometric proofs, using pictures and diagrams instead of algebraic calculations. If you know freshman calculus, an algebraic proof, which can be found in *Analytical Mechanics* by Fowles and Cassiday or our later article ``Kepler's first and third laws revisited'' is much easier.

We will also present Newton's proof of Kepler's second law in our later article "Newton's proof of Kepler's second law," and an algebraic proof in "Kinetic energy, potential energy and angular momentum in polar coordinate and Kepler's second law." In this case, Newton's proof is easier.

We will present a proof of Kepler's third law in "Newton's law of universal gravitation and Kepler's third law" in the case that the orbit of a planet is a circle, and in "Kepler's first and third laws revisited" in the case that the orbit of a planet is an ellipse.

So, with more observational and experimental evidence, ranging from an apple falling to the ground to the moon orbiting around the earth, Newton asserted that every massive object attracts another with the amount of the force being inversely proportional to their distance squared.

In other words, he showed that the law of universal gravitation explains both the orbits of the

planets in the universe and the gravitational forces on the Earth, such as an apple falling to the ground. However, everything was not that easy from the beginning. When he first tried to compare the gravitational force that attracts apple to the center of the Earth with the gravitational force that makes the Moon orbit around the Earth, he used the wrong value for the radius of the Earth. Luckily, he put this problem aside, and before publishing *Principia*, the French astronomer Jean-Félix Picard came up with a very precise value (error within 0.5% from the modern value) saving Newton.

So, let me give you some more examples which he correctly explained using his law of universal gravitation. Let's begin with "tide." Tide is the phenomenon that the sea level always changes. After the sea level reaches its highest point, it lowers and after about six hours, it reaches its lowest point, then it rises, and after about six hours it reaches its highest point and so on. So, again, that the sea level changes over time is tide. Actually, the English word "time" shares the same root as the English word "tide." In Scandinavian languages, the time is still "tid."

So, why does tide happen? See Fig. 1. The figure is not in scale, but to help your understanding it is exaggerated. The arrows denote the gravitational attraction of the Moon. You see that the water facing the moon is attracted to the Moon (denoted with "101") more than the water in the middle is attracted to the Moon (denoted with "100"). (Remember that the gravitational force is bigger if the object attracted is closer.) Thus, the water facing the Moon bulges toward the Moon. On the other hand, the water direct opposite to the Moon is less attracted to the Moon (denoted with "99") than the water in the middle is attracted (denoted with "100"). Thus, the Earth in the middle is slightly "rightward" toward the Moon compared to the water direct opposite to the Moon. The net effect on the water direct opposite to the Moon is that it is slightly "leftward." Therefore, the water direct opposite to the Moon.



Fig. 1: the tide

Fig 2: why the tide repeats

"The tidal force" is the difference between these gravitational attractions. The tidal force for the water facing the Moon in our case is "101-100=1." Similarly, the tidal force for the water direct opposite to the Moon is "99-100=-1." We see that the directions of these two forces are opposite. In other words, the tidal force "stretches" an object.

Now, more quantitative treatment of the magnitude of the tidal force. We know that the Sun's gravitational attraction of the Earth is much bigger than the Moon's gravitational attraction of the Earth. Otherwise, the Earth will orbit around the Moon instead of around the Sun. So, why did we

talk about the tidal force due to the Moon, instead of one due to the Sun? The answer is that the Moon's tidal force on the Earth is bigger than the Sun's tidal force on the Earth. Actually about double. While the Sun's gravitational attraction of the Earth is bigger, as the Sun is so far from the Earth, the differences of the gravitational attraction on different points on the Earth is relatively small, as the distances between these points are relatively small compared to the distance to the Sun. On the other hand, the Moon is so close to the Earth that the differences of the gravitational attraction on different points on the gravitational attraction on different points on the Earth are relatively large, as the distances between these points are relatively large, as the distances between these points are relatively large, as the distances between these points are relatively large, is inversely proportional to the distance cubed, instead of the distance squared as in the gravitational force. This makes the tidal force more rapidly decreasing as the distance gets farther compared to the gravitational force is quartered. As the distance to the Sun is much bigger than the distance to the Moon, the tidal force of the former is relatively less important.

So, why do we get the tides? It's because the Earth is rotating. See Fig. 2. you meet the bulged part of water every twelve hours: when you are facing the Moon directly, and when you are directly opposite to the Moon. Actually, it's not exactly twelve hours, because the Moon's position also changes as it orbits around the Earth.

Problem 1. Explain why the tide is greatest when the Moon is the full Moon and the new Moon. (Hint¹)

The calculation of the sea level difference between the high tide and the low tide due to the tidal force of the Moon was one of the three theoretical problems in the International Physics Olympiad in 1996. Luckily, the guidelines on how to approach the problem were given in detail. The answer was 54 cm. I studied this problem before I participated in International Physics Olympiad in 1998.

Sir Isaac Newton also solved the precession of the equinoxes, which had been a mystery for 2000 years. We already mentioned Hipparchus's discovery of the precession of equinoxes in the last article. Now, let me explain how Newton solved this problem. We know that the Earth is rotating. Thus, the Earth must be flattened around the North Pole and the South Pole and bulged around the equator.²

¹ First, think of the positions of the Moon and the Sun in such cases. Then, consider the combination of tidal force due to the Moon and tidal force due to the Sun. When they are acting in the same "stretching" direction, the tide will be greatest.

² This flattening and bulging is due to centrifugal force. We will learn about centrifugal force in our later article "Centrifugal force." But, this phenomenon is intuitively clear even for those who have not studied physics.

He calculated how much the bulge would be. He obtained that the equator should be farther from the center of the Earth than the North Pole or the South Pole is by 17.1 miles (27.5 km). The actual value is 13.3 miles (21.4 km). Now, as the law of universal gravitation says that all objects attract one another, the Moon attracts the Earth as well. However, as the Earth is not a perfect sphere, slightly bulged in the middle, the Moon's attraction is not uniform; it attracts the bulge more as there is more material (i.e. masses) there. So, this makes the axis of the Earth rotation spin around albeit at a very rate, as the bulge is quite small, being around 1/300 of the radius (Newton's value is 1/230). Newton calculated the rate of the precession to be 50"00"12"" (i.e., (50+0/60+12/60/60) arcseconds). This corresponds to the period of precession around 25918 years. The actual value is 25772 years.

Therefore, Newton obtained a very close value, even though he got the amount of bulge wrong. Thus, Newton's calculation was a fluke, or as some historians believe, adjusted to get the right value. Actually, he used a wrong ratio, 4.4815:1, for the tidal force of the Moon to the tidal force of the Sun, which he obtained from the data of the heights of the tides. Actually, this ratio is important because one can deduce the mass of the Moon from it, using the fact that the tidal force is inversely proportional to the cube of the distances. Then, in turn, from the mass of the Moon, one can deduce the gravitational attraction of the Moon. The correct ratio is about double, as mentioned, or precisely 2.17:1. This was not the only mistake. He assumed that the density of the interior of the Earth is uniform (which is actually the reason why he got the amount of the bulge wrong) whereas, in reality, the density becomes bigger and bigger as you go down deeper and deeper toward the center of the Earth. Moreover, he used "momentum conservation" for the place where he should have used "angular momentum conservation." You do not know yet what they are, but you will learn them in later articles.

Actually, it's not only the rotating axis of the Earth that precesses but also the axis of the Moon's revolution around the Earth does. Its full cycle takes 18.60 years. See Fig. 3. You see the side view of the Sun, the Earth, and the Moon. The dotted line denotes the plane of the Earth's orbit. The solid line denotes the plane of the Moon's orbit. The plane of the Moon's orbit is inclined about 5 degrees with respect to the plane of Earth's orbit. (We have already mentioned in "The solar eclipse and the lunar eclipse.) However, the plane of the Moon's orbit precesses while it remains to be inclined 5 degrees with respect to the plane of Earth's orbit. In other words, the direction in which the plane of the Moon's orbit is inclined changes. After half a cycle (i.e., 9.30 years), it will look like Fig. 4.



Fig. 3. the Sun, the Earth and the Moon now



Fig. 4: the Sun the Earth and the Moon 9.30 years later

Newton also calculated this and obtained 18.65 years as the period. So, he was very close. He attributed the discrepancy due to the fact that he approximated the orbit of the Moon as a circle instead of an ellipse.

Two remarks.

First, we do not know why the gravitational force is inversely proportional to the square of the distance since this is something that only God decided. Nevertheless, we can guess why he decided so. This we will explain in "The inverse square law and the 3-dimensional world."

Second, the trajectory of a planet is perfectly elliptical only when the gravitation due to the Sun is considered. In reality, the trajectory deviates away very slightly from a perfect ellipse due to the presence of gravitational attraction of other planets; the line connecting the two foci of a planet rotates very slowly rather than being fixed. See Fig. 5.





This is called "precession of perihelion." You see that the green line rotates. Actually, the figure is quite exaggerated. Otherwise, you would not have noticed the effect of precession of perihelion from the figure. In the case of Mercury, the green line rotates by 0.157 degrees per century. So, it's really tiny. However, if you calculate the gravitational effect of other planets, it should rotate by only 0.146 degrees. Einstein explained the difference. We will talk more about it later in the next article.

Further application of the Newtonian mechanics to astronomy

In 1748, the British astronomer James Bradley announced his discovery of "nutation." It is similar to the precession of the rotating axis of the Earth, but at a much smaller angle (9.2 arcseconds) and at a much shorter period (18.60 years). This is due to the change of lunar nodes just mentioned; as the orbiting plane of the moon changes, the rotating axis of the Earth changes correspondingly as we will explain shortly.

In 1749, about 20 years after Newton's death, the French mathematician, physicist and philosopher, d'Alembert published the correct solution of the precession of the equinoxes problem in his book *Recherches sur la précession des equinoxes, et sur la mutation de l'axe de la terre, dans le systême newtonien (Research on the precession of the equinoxes and on the mutation of the axis of the Earth in the Newtonian system).* It is very long, actually over 200 pages. In its introduction part, d'Alembert explained Newton's mistakes in detail. He used 2.35:1 for the tidal force ratio of the Moon to the Sun, which he deduced from the nutation discovered by Bradley. Let me explain the basic idea. If the Earth were heavier and rotated faster, then the nutation angle must be smaller, because the change of lunar nodes can only budge the Earth less fiercely. D'Alembert obtained the tidal force ratio 2.35:1 from the ratio of 5 degrees (inclination of the orbital plane of the Moon to the ecliptic) and the nutation angle 9.2 arcseconds. If you later learn the concept of "conservation"

of angular momentum," you will be able to make this statement more quantitatively. Anyhow, in 1751, Euler also obtained the correct solution for the precession problem, which was clearer and more elegant than d'Alembert's.

Another application of Newtonian mechanics is "the rotation of Moon's apsides" in the late 1740s and the early 1750s, please read our earlier article "The mathematical beauty of physics: simplicity, consistency, and unity," if you are interested.

In 1693, Halley discovered the acceleration of the moon's mean motion. In other words, on average, the time it takes for the moon to orbit around the Earth once is getting smaller, and smaller in the long run, even though it may fluctuate in the short run. As can be learned from Ptolemy's *Almagest*, Hipparchus obtained the Moon's mean angular speed from eclipses. Halley compared this value with a value he deduced from modern observations and found them to be slightly different. In 1749, Dunthorne obtained that the acceleration is 10 arcseconds per century.

In 1770, the Paris Academy proposed their prize for an investigation to see if this phenomenon could be explained by Newton's law of universal gravity. Euler won the prize, asserting that it couldn't be explained. He said, "It appears to be established by indisputable evidence that the secular inequality of the moon's mean motion cannot be produced by the forces of gravitation." In 1772, he suggested a resisting medium existed in space. In 1774, Lagrange proved that the acceleration of the moon's mean motion could not be due to the non-spherical shape of the Earth. In 1780, he went on to prove that it could not be due to the non-spherical shape of the Moon either.

In 1774, Laplace tried to solve the problem by assuming that the gravitational force is not transmitted instantly, but he failed. In 1787, he finally succeeded in explaining this phenomenon by Newton's law of universal gravity. His reasoning is as follows. The bigger the Sun's influence, the less Moon's gravity towards the Earth. The less Moon's gravity towards the Earth, the slower angular speed of the Moon (it takes more time to orbit around the Earth.) On the other hand, the sun's influence is bigger, if the eccentricity of the Earth's orbit is bigger. However, in the long run, the eccentricity of the Earth's orbit has been diminishing due to the gravitation of other planets. Thus, the Sun's influence has been decreasing, which makes the angular speed of the Moon faster. He obtained 10.3 arcseconds per century, close enough to the observed value.

However, in 1853, J. C. Adams discovered a mistake in Laplace's calculation. Laplace assumed that the eccentricity of the Earth is constant, while it changes. J. C. Adams considered this and obtained 5.7 arcseconds. Now, it is an open question and many believe that the acceleration of the Moon's mean motion is tidal friction in the seas on the Earth.

Now, to the inequalities (anomalies in the mean motion) of Jupiter and Saturn. In 1625, Kepler found

these inequalities by analyzing the motion of these planets from Ptolemy to Tycho. The orbiting periods of Jupiter and Saturn slightly changed.

Halley also found these inequalities, and in his *Tabulae astronomicae* published posthumously in 1749, he included a constant acceleration in the mean motion of Jupiter and a constant deceleration in the mean motion of Saturn. In other words, to fit with observation, he assumed that Jupiter's orbital motion gets faster at a constant rate, and Saturn's one gets smaller at a constant rate. However, in 1776, Lalande published observational evidence that these are not the cases.

In 1785, Laplace succeeded in explaining the inequalities of Jupiter and Saturn by Newton's law of gravitation. This reduced the errors of the tables from 20 arcminutes to 12 arcseconds. Laplace said, "These inequalities appeared formerly to be inexplicable by the law of gravitation-they now are one of its most striking proofs." He obtained the inequalities drawn in the figure below. The blue one is Saturn and the orange one is Jupiter.

arcminute



Fig. 6: the inequalities of Jupiter and Saturn

Notice that Halley got it wrong. For the graph around the year 1749, Jupiter really seems to accelerate (the graph is going up) and Saturn really seems to decelerate (the graph is going down). But, in the long run, they are not. They just keep repeating accelerating and decelerating periodically.

Edmond Halley

Edmond(Edmund) Halley (1656-1742) is so famous. Many of you may have heard of Halley's Comet, the first comet to be identified as a comet. Nevertheless, he has made a lot of other no less important achievements.

The British astronomer, David W. Hughes wrote that "Halley was blessed with many attributes...an

excellent brain, an avid inquisitiveness, the ability to work hard, and a rich father [in his quest to be a scientist]."³ He also wrote, "it is one thing to have a bright scientific idea, it is entirely another to be provided with the financial support to carry it out." From his father, Halley received an allowance of £300 per year, which was three times the salary Flamsteed (who measured the distance to Mars and whom Halley assisted during the former's vacation) as Astronomer Royal.

When he was an undergraduate at Oxford, Halley wrote to the Secretary of the Royal Society that he was considering going to St. Helena to catalog the Southern Stars. The Secretary and Flamsteed contacted King Charles II, who instructed the East India Company to provide Halley and his friend a free passage to the island. After returning from the island, he published the catalog of the Southern Stars. He was 22.

In 1686, Halley first established the relationship between atmospheric pressure and height above sea level.

In 1687, Halley published Newton's *Principia*. He encouraged Newton to write it up, proofread it, and paid for its printing out of his pocket.

In 1693, Halley first used rigorous statistical methods for the life insurance problem. Halley's work strongly influenced the development of actuarial science.

From Newton's *Principia*, Halley learned how to calculate the orbit of comets, and examined 24 appearances of comets in the past accordingly. He found out that 20 of them were never to be returned, one had a period of about 9000 years, and three were the appearances of comets now called "Halley's comet." They appeared in 1531, 1607 (observed by Kepler), and 1682 (observed by Halley himself). Then, he found old records of similar appearances in 1305, 1380, and 1456. In 1705, Halley concluded that they were the same comet, and predicted that it would appear at the end of 1758 or the beginning of 1759. After his death, it appeared on Christmas Day, 1758.

In 1716, Halley first correctly associated aurora with the Earth's magnetic field.

In 1718, Halley discovered the proper motion of "fixed-stars." To re-examine the rate of precession of equinoxes, he compared his measurements with those given in Ptolemy's *Almagest*. He noticed that, during the last 1800 years, three stars had changed positions, one of them as much as 30 arcminutes. Indeed "fixed stars" were not fixed. It was just that they were too far away to notice that they had changed their positions.

³ Hughes, David W. (August 1985). "Edmond Halley, Scientist" (PDF). Journal of the British Astronomical Association. London, UK: British Astronomical Association. 95 (5): 193

Finding the longitude

Galileo Galilei proposed the observation of Jupiter's moons as a method to determine the longitude.⁴ Jupiter's moons orbit around Jupiter with certain periods and certain sizes for the orbits. Thus, the time of eclipses of Jupiter's moons (i.e., Jupiter's moons invisible as they hide behind Jupiter) can be predicted quite accurately. Once eclipses are predicted and made into tables, navigators can supposedly observe the eclipses of the moons of Jupiter, and determine "the absolute time," (i.e., the time that does not depend from location to location) by comparing their observations with the tables. Let me explain what I mean. Let's say a navigator in the Pacific ocean and a navigator in the Atlantic observe Jupiter at the same time. Then, they will have the same image of the eclipses, and from the tables, the same absolute time, even though their local times are different. Once the absolute time is obtained so, navigators can measure the local time from the stars. Then, they can find the longitude by comparing it with the absolute time.

Even though predicting the eclipses was possible, but it was not easy either, as one has to consider the line of sight between the Earth and Jupiter, which is constantly changing as both orbit around the Sun; if we observe that a moon of Jupiter hides behind Jupiter, it means that it is behind Jupiter when *seen on the Earth toward the Jupiter*. To avoid complex calculations, Galileo Galilei even invented an instrument that can determine the times of eclipses of Jupiter's moons. He also invented another instrument, actually a helmet to which a telescope is attached through an eyehole, so that Jupiter's moons can be observed through the telescope on a rolling ship, but it was not successful.

When Philip III of Spain offered a huge prize for the method to find the longitude, Galileo applied for it and exchanged letters with Spain about his proposal for 16 years, but his proposal was not adopted. When the Netherlands offered a prize in 1636 for the practical method to find the longitude, Galileo applied for it. A commission was set up, and one of the Dutch commissioners tried to visit Galileo to discuss his method, but he couldn't as Galileo was under house arrest. The Netherlands lost interest when Galileo died in 1642.

Eventually, Galileo's method was never used at sea. However, about 40 years later, the Royal Academy of Sciences (in French *Académie Royale des Sciences*) successfully used Galileo's method to determine the longitude on *land*. Many tried to make platforms that remain fixed on the rolling ship so that Jupiter's moons could be calmly observed through telescopes, but they never succeeded.

In 1755, the Geman astronomer Tobias Mayer suggested the "lunar distance method." He predicted the positions of the Moon, which then can be used to determine the absolute time. After some

⁴ When I was a young child, I read in a book that Galileo proposed such a method. But, there was no explanation about the details of the method. I always wondered what it was. Now, I know it.

experiments with Mayer's tables, Great Britain adopted it, and in 1766, began publishing astronomical tables that include positions of the Sun, the Moon, and the planets, and the "lunar distances", distances from the Moon to the nearby stars.

In practice, the lunar distance method is quite complicated. The navigator has to take into account the parallax of the Moon; at different locations on the Earth, the apparent positions of the Moon are different. Also, the changing size of the Moon had to be taken into account, because the distances to the nearby stars are measured from the edge of the Moon while the lunar distances in the table are given by the distances from the center of the Moon. The changing size, which is due to the changing distance from the Moon to the Earth, was also provided by the tables. As we briefly mentioned earlier, the lunar distance method was replaced by the method based on accurate, affordable clock in the middle of the 19th century. However, Great Britain kept publishing the tables for the lunar distances until 1906.

Discovery of new planets

In 1781, while observing stars at night, Sir William Herschel, a German-British astronomer, and composer noticed that one of them seemed to be larger than the stars near it. Suspecting that it might be a comet, he increased the magnifying power from 227 to 460 and 932, because he knew that the fixed stars did not increase their size in the same ratio as the magnifying power. With these great powers, he observed that the size of the "comet" increased in the same ratio, and it "appeared hazy and ill-defined" whereas the fixed stars "preserved that lustre and distinctness." So, he was convinced that it was a "comet," and not surprisingly, it changed its position on the following nights.

Many astronomers tried to calculate the orbit of the comet to determine whether the orbit is parabolic, meaning that the comet is indeed a comet and will never return, or the orbit is elliptic, meaning that the comet is actually a planet. (To learn about parabola and ellipse, read our earlier article "Conic sections.") However, they could not determine it due to the big size of the orbit. Then, Anders Lexell, a Finnish-Swedish astronomer, found the record of a star observed in 1759 by Christian Mayer. Assuming that this record was actually about the "comet," he found out in 1783 that the "comet" is actually a planet, as its orbit is elliptic. Actually, it was found that the planet had been previously observed 19 times since 1690, but astronomers thought that it was a fixed star. This is the first planet to be discovered since ancient times. The discoverer of the new planet, Herschel named it "Georgium Sidus" after King George III. After much negotiations, King George III hired Herschel as a personal astronomer with an annual stipend of £200, a not big amount, but his only duty was that he moved to Windsor so that the Royal Family could look through his telescopes anytime.

But, as non-British astronomers didn't like the name "Georgium Sidus," the name didn't stick. The

German astronomer Johann Elert Bode suggested calling it "Uranus" arguing that just as Saturn was the father of Jupiter, the new planet should be named after the father of Saturn. (Jupiter, Saturn, Uranus are gods in Greek-Roman mythology.) In the early 19th century, the names "Georgium Sidus," "Herschel," "Uranus" were simultaneously used. However, since 1850, the British Nautical Almanac uses Uranus. Now, it is exclusively called Uranus.

In 1790 Jean Baptiste Joseph Delambre, a French astronomer published tables to compute the positions of Uranus based on its observations made since 1690. The orbit of Uranus indeed seemed to agree with the Newtonian predictions when the gravity due to other planets was accounted. However, in 1820, Alexis Bouvard, a French astronomer found some discrepancies; he could fit the orbits of Jupiter and Saturn with the Newtonian predictions, but he could not do so with the orbit of Uranus. The discrepancy was as big as 50 arcseconds, which was much bigger than 5 arcseconds, the observable accuracies at the time. Therefore, he rejected the older observations of Uranus suggesting "some foreign and unperceived cause which may have been acting upon the planet." Still, within more recent data, he could fit the orbit of Uranus only within 9 arcseconds. However, even this discrepancy grew. In 1841, it grew as big as 70 arcseconds.

Some even suggested that Newton's law of gravity didn't apply beyond Saturn. But, others believed in Newton's law of gravity. In 1843, John Couch Adams, who just graduated from Cambridge as the top mathematics undergraduate, began to tackle this problem. By October 1843, he showed that a presence of a new planet lying beyond Uranus had the potential to explain the orbit of Uranus. In 1844, assuming a circular orbit, he obtained an approximate orbit of the new planet and asked Prof. Challis to obtain the latest observations of Uranus, which the Astronomer Royal, Airy supplied. Then, he calculated everything again, based on the new data, with more precisions. In September 1845, he provided Challis with the predicted position of the new planet for September 30th, 1845. On September 22nd, Challis wrote to Airy about the predicted positions expressing his confidence in Adams's capabilities. Adams visited Airy on his way to his own family in Cornwall, but Airy was in Paris. On October 21st, Adams visited Airy again on his way back from Cornwall, but Airy was in London. He visited Airy's home later in the day, but as Airy was at dinner, his butler turned him down. So, Adams left his detailed results showing that he reduced the discrepancies of the orbit of Uranus to about 1 arcsecond by assuming the existence of a new planet. Of course, he also provided his prediction of the position of the new planet. We now know that his prediction of the position was correct within 2 degrees.

On November 5th, Airy wrote to Adams asking about the perturbation of the radius vector of Uranus. This requires some explanations. Note that there can be two types of discrepancies in the orbit: tangential and radial. See Fig. 7.



Fig. 7: the radial discrepancy and the tangential discrepancy

A is the position where Uranus would have been located if the new planet didn't exist. B is the actual position of Uranus, the one affected by the new planet. Notice that the difference between the two positions is the combination of Δ rad (radial discrepancy) and Δ tan (tangential discrepancy), which I denoted with two dotted arrows. Of course, the figure is very exaggerated. The actual discrepancies (called "perturbations") are much smaller.

The tangential discrepancies are much easier to detect than the radial discrepancies. If there are tangential discrepancies, one can immediately notice them because the apparent position significantly changes. On the other hand, the radial discrepancies hardly matter for the apparent position, as the line of sight from Uranus to the Earth is almost the same as the radial direction as the orbit of Uranus is almost 20 times bigger than the one of the Earth; imagine you connect a line from Uranus to the Earth. It will be almost parallel to the radial direction. Of course, if there are radial discrepancies, they will be also manifested by its apparent size. For example, if an object is slightly farther, it will look slightly smaller. But, considering that Uranus looks so small anyway, as it is so far away, it is impossible to deduce the radial discrepancies from the very small change of its apparent size.

So, Airy was asking whether Adams's suggestion of the new planet that correctly corrected the longitude also corrected the radial vector. However, as I just explained, as the radial discrepancies are very hard to measure affecting very little in the apparent position, it is virtually impossible to check that the radial vectors are corrected correctly. Adams never replied. However, as a draft dated November 13th, 1845 was discovered, we now know that he started writing the reply. In the draft he never sent, he apologized for visiting Airy at an inconvenient time, but he had no choice because he had to get back to Cambridge on the same day. Then, he wrote, "The paper I then left contained merely a statement of the results of my calculation." According to Airy's later recollection next year

(1846), he had "waited with much anxiety" for Adams's reply. But, not hearing back from Adams "not only left the matter in an unsatisfactory state... but also stopped me from writing again."

While Adams was not writing a reply to Airy, Urbain-Jean-Joseph Le Verrier, a French astronomer worked on solving the problem of Uranus by proposing a new planet, not knowing about Adams's work. In June 1846, Le Verrier announced his prediction of the position of the new planet. At the end of June, Airy sent a letter to Le Verrier asking about the radial vector. Unlike Adams, Airy replied immediately saying that his theory accounted for the perturbation in the radial vector. As Le Verrier's prediction on the position agreed with Adams's one within 1 degree, on July 9th Airy asked Challis to look for the new planet with the Northumberland equatorial telescope at Cambridge. (An equatorial telescope can be rotated around an axis parallel to the axis of Earth rotation so that the rotation of the Earth can be easily compensated.) On July 29^{th,} 1846, the search began. Challis observed and made a catalog of all stars near the predicted position of the new planet. On August 31st, 1846, Le Verrier published the details of the planet. As the French astronomers were not quite interested, he sent his results to Galle at the Berlin Observatory on September 18th, 1846 by a letter, which arrived on the morning of September 23rd, 1846. Luckily, earlier that year, a German astronomer Bremiker completed a catalog of all stars in the part of the sky that included the predicted position of the new planet. Galle's student Heinrich Louis d'Arrest suggested Galle using this catalog. At the very night of the day he received the letter from Le Verrier, Galle began to observe stars in the relevant part of the sky and called their positions to d'Arrest, who checked whether they were on the catalog. The new planet was found within an hour. The new planet is now called "Neptune."

Actually, only after the announcement of its discovery was it apparent that Challis had also observed Neptune on July 30th, 1846, and at a different position on August 12th, 1846 but he hadn't realized that it was not a fixed star. By the time the news of the discovery reached Cambridge in late September, Challis cataloged 3150 stars.

On November 13, 1846, at the Royal Astronomical Society meeting, Airy recalled that he regarded "whether the error (i.e. perturbation) of the radius vector would be explained by the same theory which explained the error of longitude" an *experimentum crucis*, if the explanation failed, he thought, it would mean a failure in Newton's law of gravitation.

To this, Adams replied, without apologies, after explaining some technical calculations of the orbit, "the corresponding increase in the radius vector would not be very different from that given by my actual theory."

On November 30, 1846, the Royal Society awarded the Copley Medal to Le Verrier. The citation didn't mention Adams. Nevertheless, Adams didn't denounce Challis or Airy, and acknowledged Le

Verrier's credit to the discovery of Neptune, acknowledging his own failure to convince astronomers of his predictions.

Foucault pendulum and the rotation of the Earth

Many dumb people, especially flat earth believers argue that the Earth is not moving because we would have noticed it otherwise. They are wrong for a couple of reasons. First, it is not easy to notice that the Earth is moving for the same reason as it is not easy to notice that the airplane you are on is actually flying if there is no air turbulence, and if you do not see outside the window. Actually, seeing-outside-the-window-option is also available in the case of the Earth. We have already explained that the stars are constantly rotating around us, approximately once every 24 hours (actually 23 hours 56 minutes 4 seconds). (We will explain the reason why it's not exactly 24 hours shortly.) Second, the rotation of the Earth does affect the motion of things on the Earth, even though it is not easy to notice it without careful scientific instruments. The first such an observation was made by Léon Foucault in 1851.





Fig. 8: a Foucault pendulum above the North PoleFig. 9: the difference between the rotating
period of the Earth and a day

See Fig. 8. A pendulum is right above the North Pole. Unless somebody is shoving or twisting the pendulum, it will oscillate in the same oscillating plane (i.e., same direction). However, the Earth is rotating counter-clockwise. Therefore, from the point of view of someone on the Earth, the oscillating plane of the pendulum seems to rotate the other way around, i.e., clockwise. As the Earth rotates once every 23 hours 56 minutes 4 seconds, the oscillating plane will also rotate once every 23 hours 56 minutes 4 seconds, the oscillating plane will also rotate once every 23 hours 56 minutes 4 seconds. You know that a day is 24 hours, not 23 hours 56 minutes 4 seconds. It's because a day is based on the time it takes between a noon and the next noon. See Fig. 9. It takes 23 hours 56 minutes and 4 seconds for point A to rotate into point C. This is the period of rotation for the Earth. However, the Earth orbits around the Sun meanwhile. Therefore, point C is no longer a noon, but point B is. It takes about 4 minutes for point

C to rotate into point B. In other words, it takes 24 hours for point A to rotate into the point B. Think about it. The 4 minute difference arises because the direction that the Earth is facing the Sun changes as the position of the Earth relative to the Sun changes. The fixed stars and Foucault pendulum cannot know which direction the Earth is facing the Sun. So, 23 hours 56 minutes is the true period of rotation for the Earth.

Anyhow, such a pendulum that can be used to detect the rotation of the Earth by the rotation of its oscillating plane is called the "Foucault pendulum." What will happen to a Foucault pendulum at the South Pole? The Foucault pendulum rotates anti-clockwise, as the Earth rotates clockwise once every 23 hours 56 minutes (=23.93 hours). In other words, the oscillating plane rotates about 15(=360/23.93 hours) degrees per hour in an anti-clockwise direction. At other points on the Earth, it is somewhere between these two cases. On the Northern hemisphere, it rotates clockwise, and on the Southern hemisphere, it rotates anti-clockwise. At the equator, it can rotate neither clockwise nor anti-clockwise, so the oscillation plane does not rotate at all; it rotates 0 degrees per hour. Therefore, in the Northern hemisphere, the oscillation plane rotates clockwise with a certain angle per hour, which is 0 between 15 degrees, which depends on the latitude of the position. In other words, in the Northern hemisphere, it takes 23 hours 56 minutes or more for the oscillating plane to come back to the original plane. The same can be said about a Foucault plane in the Southern hemisphere. Only the rotation direction is the opposite.

The Milky Way, or our Galaxy

We have briefly mentioned that Galileo had discovered that the Milky Way was a group of many stars. The Milky Way stretches across the sky, even though most urban residents never noticed the Milky Way, as the night is too bright in cities. See the photograph of the Milky Way taken in Paranal Observatory in northern Chile.



Fig. 10: a photo of the Milky Way taken in Chile, by European Southern Observatory astronomer Yuri Beletsky

So, why are stars stretched in the sky like this? We now know that our Solar system is located in a group of stars called "our galaxy" also called "the Milky Way." The Milky Way we see in the night sky is the view we have when we look toward the center of our galaxy. I said, "our galaxy" instead of "the galaxy" which means that there are other galaxies as well. A "galaxy" is a group of stars located closely. Here is a picture of a typical galaxy.



Fig. 11: the galaxy NGC 1414 imaged by the Hubble Space Telescope.

You see that it looks like a disc and has spiral arms. Such a galaxy is called a "spiral galaxy." It is rotating. Below, you see a photograph of another galaxy, which is seen as more "side-view" on the Earth.



Fig. 12: the image of galaxy NGC 4565. Photo from ESO (European Southern Observatory)

Of course, I would have liked to include pictures of our galaxy taken from both a "top" view and a "side" view, but it is impossible to take selfies of our galaxy because we cannot manufacture a long enough selfie-stick. Nor is it possible to take pictures of the same galaxy, one from a top view and another from a side view, because it's impossible to travel such a great distance to take pictures from different angles.

As it was not certain that the Milky Way was just a group of stars, it was not certain either that these galaxies were groups of stars, because they just looked like "clouds" in low-resolution telescopes.

In 1781, the French astronomer Charles Messier published a catalog of 103 such "nebulae" (meaning "clouds" in Latin), as they could be easily mistaken for comets during comet huntings. This catalog is now called the "Messier catalog" and his classification is still used. For example, M31 is what we now call the "Andromeda galaxy." We now know that some of these "nebulae" are gas clouds in our galaxy, and others are galaxies.

Sir William Hershel, the discoverer of Uranus, was the most important pioneer in the idea that our Solar system is in our galaxy, the Milky Way. Assuming that stars are equally scattered and equally bright, and the only reason that we see some as brighter and darker than the others are that they are located closer and farther than the others, he estimated the (relative) distances to more than 3000 stars in his papers published in 1784 and 1785. (Remember that it was before the time when the first annual parallax was measured.) He obtained that the distribution of the stars was in the form of a disc. It extended in the plane of the Milky Way over 800 times, and the thickness 150 times the mean distance of two stars, supposed to be the distance from Sirius to the Sun. He also concluded that the Sun was very close to the center of our galaxy. That our galaxy is in the form of the disc is correct. Nevertheless, we now know that neither Hershel's assumptions nor his conclusion were correct. He also observed other "nebulae" which he called "telescopic milky-ways" and which we now identify as galaxies. He wrote, "the stupendous sidereal system we inhabit...consisting of many millions of stars, is, in all probability, a detached Nebula."

Earlier in 1783, Hershel also found out that the fixed stars tended to move toward a certain point in the sky. He correctly interpreted that this was due to the motion of the Sun and the Solar System; as we are moving, the other fixed stars seem to move in the opposite direction.

In 1927, the Dutch astronomer Jan Hendrik Oort discovered that our galaxy rotated itself and that the Sun was not at its center, but 30,000 light years away from it. He deduced this from the relation of the distance to the stars and their radial velocities. I omit the actual details of his deduction because I could not understand it myself due to their complexities.

Cepheid variables and the distance determination

Regarding the methods to determine the astronomical distances, so far we have only talked about the parallax. However, the method of parallax has some limitations. Even with modern technology, some astronomical objects are placed too far away to measure a noticeable parallax. However, astronomers figured out new methods to measure such far distances. In this section, we will talk about the first of such methods. Before that, we need to talk about the previous history leading to it.

Earlier, we have seen that some planets such as Mars change their brightness, as they get closer and farther from the Earth. Then, you might ask: are there any *fixed* stars that change their brightness as well? There are. They are called "variable stars." They change their brightness with a fixed period.

The first variable star to be recognized as one is Algol. Historians say that an ancient Egyptian calendar of lucky and unlucky days is the evidence that Egyptians knew about the variability of Algol; the period of the lucky and unlucky days in the Egyptian calendar coincided with the period of the brightness of Algol, 2.85 days. The Italian astronomer Geminiano Montanari re-discovered the variability of Algol in 1667, but he did not know that it was periodic. The British amateur astronomer John Goodricke discovered the periodic nature of the variability of Algol and presented his findings to the Royal Society in 1783 at the age of 18. He also correctly suggested that the periodicity was caused by a dark body and another bright star passing each other periodically. This was proven correct in 1889 when Vogel and Scheiner found out from the Doppler shifts that Algol got darker when receding (i.e. moving away) from us and it got brighter when approaching us. The obvious and correct interpretation is that the visible star Algol is going behind a dark body when it moves away from us, and is emerging from a dark body when it approaches us.

However, there are other types of variable stars now called "Cepheid variables" or "Cepheids." In September 1784, Goodricke's collaborator and mentor Edward Pigott discovered the first Cepheid variable, Eta Aquilae. A few months later, Goodricke found another Cepheid variable, Delta Cephei, which "Cepheid variable" is named after. When the Doppler shifts of Delta Cephei were first measured in the late 19th century and the early 20th century, there was indeed the periodicity in the radial velocity of Delta Cephei. At first, it seemed that Goodricke's explanation for variable stars also worked. However, it was hard to relate the radial velocity with the changing brightness for the following reason. Cepheids such as Delta Cephei have a common tendency: they get bright very quickly, then they get dark very slowly, then they repeat this pattern. If Goodricke's explanation for variable stars were meant to work, they should get bright and dark at an equal pace.

In 1914, Harlow Shapley successfully defended his new theory: the receding and approaching of

Cepheids were due to their actual expansion and contraction. When they expand they are getting hotter and brighter, and when they contract they are getting colder and darker. Unlike the Sun, which doesn't change its brightness periodically, Cepheids are not at equilibrium. This idea was accepted by the astronomical community when Eddington came up with a theory of pulsating stars in 1918.

However, before the mechanism behind the variability of Cepheids was established, the American astronomer Henrietta Leavitt (1868-1921) achieved a real breakthrough in 1912, which would turn out crucial in the determination of astronomical distance, as we will see shortly.

To study the humanities, Leavitt entered Radcliffe College, a women's college that later merged with Harvard. It was not until her senior year that she took her first course on astronomy; she fell in love with astronomy. After finishing her degree, she stayed there to take graduate-level courses on astronomy and to work as a graduate student at the Harvard Observatory. In 1902, she received a permanent position there and worked under Pickering, who set a project to measure the brightness of all known stars. Pickering assigned Leavitt to measure the brightness of variable stars.

Going beyond this original assignment, she tried to find the relation between the brightness of a Cepheid and its period. After all, there could be a certain unknown relation. However, there was a big hurdle: some Cepheids are placed farther away from the Earth than the others, meaning that they would seem darker on the Earth, not because of their certain intrinsic property from which a certain relation with their periods could be deduced, but because they are simply farther away; the apparent brightness of a star depends on the distance to the star. What she wanted and needed to find was a relation between the absolute brightness (i.e. the inherent brightness of a star that is independent of the distance from us) and the period.

Then, she came up with a good idea. How about measuring the brightness of Cepheids that are located roughly the same distance from us? She considered the stars in the Large Magellanic Cloud (LMC) and the Small Magellanic Cloud (SMC). LMC and SMC are small stellar systems, orbiting very near the Milky Way. She examined 1777 variable stars in LMC and SMC and identified 47 as Cepheids. Now, if a Cepheid in SMC seems to be brighter than another in SMC, it is not because the former is closer to us than the latter, as they are both located at roughly the same position, namely, where SMC is located. Rather, it would mean that the former is intrinsically brighter than the latter. She could now compare the brightness of Cepheids objectively. What she found was very interesting. Brighter Cepheids had longer periods. On the left side of Fig. 13, you see a graph from Leavitt and Pickering's 1912 paper "Periods of 25 variable stars in the Small Magellanic Cloud." The x-axis is the period of Cepheids expressed in days, and the y-axis is their brightness expressed in the magnitudes. (The brighter a star, the bigger its magnitude.) The upper graph is the maximum brightness and the lower graph is the minimum brightness. This relation is now called "the period–luminosity relation."

The right graph is the same as the left one, but with a different scale, so that the relation looks more clearly.



Fig. 13: The period-luminosity relation from Leavitt and Pickering's 1912 paper

Then, there was another big breakthrough by the Danish astronomer Ejnar Hertzsprung (1873-1967) in 1913. Remember that our Sun, therefore, also the Solar System are moving. This means that the apparent positions of fixed stars change. Per a century, the Solar System moves 420 times the distance between the Sun and the Earth. Therefore, if we compare the apparent position of a fixed star now and a century ago, we can measure the distance to it, even some of the ones whose annual parallax is too small to be measured, as the parallax is now 420 times bigger. Of course, this method is not totally correct, because not only the Solar system but also the fixed stars move. To make the matter worse, we do not know the exact velocity of fixed stars. Therefore, assuming that they are moving in random directions, Hertzsprung averaged the apparent motions of a group of 13 Cepheids nearby us, so that their random motions can be approximately eliminated. After this elimination he used the parallax to determine the average distance to the 13 Cepheids; while he could not figure out the distance to each of them, he could obtain meaningful data from them. This method of the determination of distance by Hertzsprung is called "statistical parallax." These 13 Cepheids were too far away to determine the distance from annual parallax.

Hertzsprung also assumed that Cepheids were what we now call "standard candles," astronomical objects that allow us to measure distances because they have the same absolute brightness. Let me explain what I mean. Being standard candles, if two Cepheids "A" and "B" have the same period, they have the same absolute brightness. If A seems fainter than B, it is because A is farther away than B. Then, by measuring how many times A is fainter than B, we can estimate how many times A is farther away than B. Thus, if we know the distance to B, by using a certain method such as statistical parallax, we can also estimate the distance to A.

Combining the average distance to the 13 Cepheids with their apparent luminosity, he determined the absolute luminosity of Cepheids. Then, from Leavitt's data on the apparent luminosity of Cepheids in SMC, he could determine the distance to SMC. If he had calculated correctly with his data, he should have obtained 30,000 light-years. However, he made a mistake when writing down the value, so he omitted one "0" and wrote, 3,000 light-years. In 1914, Harlow Shapley, an American astronomer, used more Cepheids and revised the distance to SMC to 95,000 light-years. There were some errors in the data on the apparent brightness of the faint southern stars. We will also talk about other standard candles later.

Is Our Galaxy the only galaxy in our Universe?

Harlow Shapley, who we just mentioned, and who correctly defended the pulsation theory of Cepheids, as we have mentioned earlier, argued that our galaxy was the only galaxy in our universe.

Harlow Shapley was born in 1885. To save money for college, he worked as a crime reporter and a police reporter from the age of 15. After finishing a six-year high school program as a valedictorian in one and a half years, Shapley entered the University of Missouri at Columbia to study journalism at the age of 20. Finding that the journalism department wasn't open yet, he opened an alphabetical order of course catalog and found "archaeology" at the top of the list. According to him, he could not pronounce "archaeology," so he tried the subject next on the list, namely, astronomy. After four years, he received B.A., and the next year M.A. Then, he went to Princeton and after two years, he received Ph.D. in 1913. Then, he went to the Mount Wilson Observatory in California, and devoted himself to the study of Cepheids.

So, why did Shapley think that our galaxy was the only one? He argued that our galaxy was so big that our universe could not be much bigger; it had to be the only one in our universe.

On the other hand, Heber Curtis argued that the distance measurement based on the Cepheids couldn't be reliable. Thus, he argued that our galaxy was much smaller than Shapley's estimate, and accordingly, there could be many other galaxies in our universe.

Shapley argued back. He mentioned a recent nova in Andromeda "nebula," which he had discovered in 1917. A nova (meaning "new" in Latin) is a star that suddenly gets enormously bright and fades over in several weeks or several months. The nova he had observed was brighter than the rest of the entire Andromeda "nebula." He argued, if Andromeda "nebula" were a galaxy with its size comparable to our Galaxy's, the nova would have to be unbelievably bright, brighter than our Galaxy, which we now know to have hundreds of billions of stars like the Sun. That a nova can emit such an enormous of energy in a short time is unlikely. He concluded that Andromeda "nebula" was just a gas cloud in our galaxy. On the other hand, Curtis pointed out that there were more novae in Andromeda "nebula" than our galaxy; it is unlikely that a small section of our galaxy has more novae than our entire galaxy.

The National Academy of Sciences invited Shapley and Curtis and held an event called "the Great Debate" on April 26^{th,} 1920. The two astronomers had a public debate in front of the audiences including Einstein who was having a tour in America after becoming famous by Eddington's solar eclipse report, which we will talk about in the next article.

There is an interesting anecdote that shows how fervidly Shapley supported his position. Just after the Great Debate, Milton Humason, Shapley's protege, showed him photographs of stars that he had found in Andromeda "nebula," suggesting that they were probably Cepheids outside the Milky Way. Shapley explained that they couldn't be stars because Andromeda "nebula" was just a gas cloud. He took his handkerchief from his pocket and wiped the identifying marks off of the back of the photographic plate.

Of course, Shapley was wrong. As mentioned, we now know that there are galaxies other than our galaxy. Moreover, the "nova" in Andromeda galaxy was indeed enormously bright. While the "nova" in Andromeda galaxy was too far away to be visible during the day, we know from historical records that some "novae" in our Galaxy were bright enough to be visible during the day, such as the supernova "SN 1054," observed in 1054. On the other hand, Curtis was not entirely correct either. Our galaxy is not that small.

In the next article, we will see how this debate was concluded by the American astronomer, Hubble. Also, we will explain what supernovae are, and why they were crucial in discovering that our Universe is expanding faster and faster.

Summary

- The size of the solar system is determined by measuring the parallax within different locations of the Earth. Transits, the phenomenon that Mercury or Venus pass across the Sun, give good opportunities to measure such parallax accurately.
- Newton mathematically showed the orbit of any planets around the Sun should be a conic section with one of the foci located at the Sun if the Sun's gravitational force on the planet is directed right toward the Sun and inversely proportional to the square of the distance between the planet and the Sun.
- Newton, thus, explained Kepler's first law. With more observational and experimental evidence, ranging from an apple falling to the ground to the moon orbiting around the

earth, he asserted that every massive object attracts another with the amount of the force being inversely proportional to their distance squared.

- Newton explained the tide, the sea level rising and lowering. Even though Sun's gravitational attraction on the Earth is bigger than Moon's gravitational attraction on the Earth, Moon's effect on tide is bigger than Sun's effect because the effect on the tide is inversely proportional to the distance cubed while the gravitational attraction is inversely proportional to the distance squared.
- Newton correctly attributed to the precession of equinox to the fact that Moon's attraction on the Earth is not uniform, because the Earth slightly bulges around the equator.
- After learning how to calculate the orbit of comets from Newton's *Principia*, Halley carefully examined the past record of comets. He found out that the same comet had appeared periodically in the past. Thus, he was able to predict when the comet would appear next time. The comet indeed appeared at the predicted time. This comet is now called "Halley's comet."
- Uranus's orbit seemed to deviate from the prediction of Newtonian gravity. Based on the deviation, a new planet, which affected Uranus's orbit, was predicted. The new planet was found at the predicted location. The new planet is Neptune.
- By Foucault's pendulum, one can know that the Earth is rotating. Its rotation direction in the Southern hemisphere is opposite of the one in the Northern hemisphere, and it doesn't rotate at all on the equator.
- A true rotation period of the Earth is a little bit shorter than 24 hours. It's because the rotation period of the Earth is measured relative to fixed stars, while a day is measured relative to the Sun. The difference comes from the fact that the position of the Earth relative to the Sun changes while it rotates around itself.
- The Milky Way that stretches the night sky is the view we have when we look toward the center of our galaxy.
- "Standard candles" are astronomical objects that have certain fixed absolute brightness which allow us to obtain the distances to them by measuring the apparent brightness seen here on the Earth. If they seem very bright here, they are close. If they seem very dark here, they are far away.

Fig. 5 is adopted from https://en.wikipedia.org/wiki/File:Relativistic_precession.svg

Fig. 8 is from

https://commons.wikimedia.org/wiki/File:Foucault_pendulum_at_north_pole_accurate.PNG

Fig. 10 is from

https://web.archive.org/web/20081121184421/http://www.eso.org/gallery/v/ESOPIA/Paranal/phot-33a-07.tif.html or https://commons.wikimedia.org/wiki/File:ESO-VLT-Laser-phot-33a-07.jpg

Fig. 11 is from

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Fig. 12 is from

https://commons.wikimedia.org/wiki/File:NGC_4565.jpg

Fig. 13 is from

Miss Leavitt in Pickering, Edward C. "Periods of 25 Variable Stars in the Small Magellanic Cloud" Harvard College Observatory Circular 173 (1912)