Bosons, Fermions and the statistical properties of identical particles

All macroscopic objects are distinguishable, even though they may be alike. You can distinguish identical twins from one another by making them wear different T-shirts. Billiard balls may look alike, but you can distinguish them by writing numbers on them.

However, this does not apply to microscopic objects. You cannot distinguish electrons from one another because you cannot write numbers on them. We say that they are "identical" or "indistinguishable."

When I say that they are indistinguishable, I am serious. The statistical properties manifest it. To ease your understanding, let's start with distinguishable particles. Suppose there are two states 1 and 2. And let's say that two distinguishable particles A and B can occupy them with equal probabilities for both 1 and 2. There are four cases:

1) A occupying 1, B occupying 1
2) A occupying 1, B occupying 2
3) A occupying 2, B occupying 1
4) A occupying 2, B occupying 2

Each case is equally likely with 25% probability.

However, unintuitive things happen when we let two identical bosons "C" and "C" occupy the states 1 and 2. For simplicity, we will assume that both states 1 and 2 are equally likely as in the previous example. Then there are only three cases:

1) C occupying 1, C occupying 1
2) C occupying 1, C occupying 2
3) C occupying 2, C occupying 2

Each case is equally likely with 1/3 (33.33...%) probability. Notice that we cannot distinguish between "C occupying 1, C occupying 2" and "C occupying 2, C occupying 1," making the case different from that of distinguishable particles.

Finally, let's consider the case in which two identical fermions "D" and "D" occupy the states 1 and 2. In an earlier article, I explained that when we have more than one identical fermion, they cannot occupy the same state. So there is only one possible case:

1) D occupying 1, D occupying 2

This is because it is not possible for two Ds to occupy state 1 or two Ds to occupy state 2.

Junior physics majors learn these amazing and interesting stuff, but I thought that the readers might not want to wait to become one.

Problem 1. Suppose there are two states 1, and 2. And let's say that three identical bosons "E," "E" and "E" occupy these states. If there is an equal chance that E occupies state 1 or state 2, what is the probability that all these three identical bosons occupy state 1? (i.e. none occupies state 2) In our later article "The Bose-Einstein distribution, the Fermi-Dirac distribution and the Maxwell distribution," we will systematically solve this problem in a general case in which we have an arbitrary number of states and an arbitrary number of identical particles. The solution in the general case was crucial to my research on black hole. We will see why it was crucial in our later articles "Quantum corrections to Hawking radiation spectrum," and "Maxwell-Boltzmann type Hawking radiation." For laymen, I reviewed them in "Discrete area spectrum and the Hawking radiation spectrum II: Single unit area deduction" and "Discrete area spectrum and the Hawking radiation spectrum III: Maxwell-Boltzmann."

Summary

- The concept of "identical" particles leads to unintuitive statistics.