Identity matrix

In the last article, I introduced matrices and linear algebra, in which I used the "black box" that satisfies two conditions. Something like a black box is called a "linear map." In this article, I will introduce the identity matrix which is a very simple linear map.

Think of the black box that outputs the numbers that were inputted.

For example, if you enter (1, 0, 2, 3), it spits out the same numbers, namely, (1, 0, 2, 3). Such a linear map, or a black box is called "the identity map." We can write this condition of the identity map "I" mathematically as follows:

$$I\left(x_1, x_2, x_3, x_4\right) = \left(x_1, x_2, x_3, x_4\right) \tag{1}$$

Let's check that this black box is a linear map. (i.e. it should satisfy the two conditions that I mentioned in the earlier article.) You can easily check this as follows:

$$I\left(n\ x_1, n\ x_2, n\ x_3, n\ x_4\right) = \left(n\ x_1, n\ x_2, n\ x_3, n\ x_4\right) \tag{2}$$

So the first condition is satisfied. The second condition is also satisfied as,

$$\begin{aligned} I\left(x_1+y_1, x_2+y_2, x_3+y_3, x_4+y_4\right) &= \left(x_1, x_2, x_3, x_4\right) + \left(y_1, y_2, y_3, y_4\right) \\ &= \left(x_1+y_1, x_2+y_2, x_3+y_3, x_4+y_4\right) \end{aligned} \tag{3}$$

Now, the natural question one may ask is: "How can we represent this linear map as a matrix. The answer is the following:

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

So the diagonal part is always 1, and the off-diagonal part is always zero. Let's check that the above matrix, which is called the identity matrix, really functions as the identity map. This is easy, as

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \tag{5}$$

So, it indeed gives the same numbers as entered.

Notice that this identity matrix works only when four numbers are entered, as this identity matrix is a 4×4 matrix. Notice also that this matrix has the property that the number of rows and the number of columns are the same, namely 4. This is natural since an identity matrix spits out the same number of outputs as the inputs. In other words, if a matrix spits out 5 numbers, even though you have entered only 4 numbers, it can't be an identity matrix, since it has to spit out an extra number which was never entered. Similarly, if a matrix spits out only 3 numbers, even though you have entered 4 numbers, it can't be an identity matrix, since the last number you entered doesn't come out of the black box, and just disappears.

So far, we have only considered the identity matrix in which four numbers are entered and four numbers are spit out. In other words, we have only considered the 4×4 identity matrix. However, there is no reason that only 4 is allowed as the size of the identity matrix. In general, there can be other sizes of identity matrices. For example, 1×1 identity matrix, 2×2 identity matrix and 3×3 identity matrix look like the following:

$$\begin{bmatrix} 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Respectively, the 1×1 identity matrix accepts one number and spits out the same number. The 2×2 identity matrix accepts two numbers and spits out the same two numbers. Similarly, an $n \times n$ identity matrix accepts n numbers and spits out the same n numbers.

You may easily get a sense of what $n \times n$ identity matrix looks like. Again, the diagonal part is always 1, while the off-diagonal part is always zero.

Summary

- An identity matrix I is a $n \times n$ matrix that satisfies $AI = A$ and $IA = A$ for any matrix.
- An identity matrix has 1s in diagonal part and 0s in the off-diagonal part.