Implicit differentiation

There are two kinds of functions: explicit functions and implicit functions. You are already familiar with explicit functions. If you describe a quantity explicitly in terms of other variables, that is an explicit function. For example, if you want to express y in terms of an explicit function of x, something like $y = f(x)$ for a suitable expression $f(x)$ will do the job. On the other hand, if you describe a quantity not explicitly in terms of other variables but in terms of equations that the quantity and the variables satisfy, the quantity is expressed in terms of implicit function of other variables. For example, if you want to express y as an implicit function of x, $g(x, y) = 0$ will do the job.

You already know how to differentiate explicit functions. In this article, we will teach you how to differentiate implicit functions. It is easily done with examples.

Let's say we have the following equation.

$$x^3 + x^2y + xy^2 - y^3 + 1 = 0 \tag{1}$$

Then, what is dy/dx when $x = 1, y = 2$? First, you need to check that the problem makes sense; when $x = 1, y = 2$, the right-hand side of (1) is indeed 0. Now, let's differentiate (1) with respect to x. We get:

$$3x^2 + 2xy + x^2\frac{dy}{dx} + y^2 + x \cdot 2y\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0 \tag{2}$$

$$\frac{dy}{dx} = \frac{3x^2 + 2xy + y^2}{3y^2 - 2xy - x^2} = \frac{3 + 4 + 4}{12 - 4 - 1} = \frac{11}{7} \tag{3}$$

This is the answer.

Careful reader may notice that we have already dealt with an example of implicit differentiation when we differentiated $x = \ln y$ in our earlier article "Exponential function and natural log." Let me repeat it here. $x = \ln y$ implies $y - e^x = 0$. Differentiating this with respect to y, we have

$$1 - e^x\frac{dx}{dy} = 0 \tag{4}$$

$$\frac{dx}{dy} = \frac{1}{e^x} = \frac{1}{y} \tag{5}$$

Problem 1. Let's say we have

$$x^3 + \sin x + y^2 + y^3 = 2 \tag{6}$$

what is dy/dx and dx/dy when $x = 0, y = 1$?

Summary

- If you describe a quantity explicitly in terms of other variables, that is an explicit function.
- If you describe a quantity not explicitly in terms of other variables but in terms of equations that the quantity and the variables satisfy, the quantity is expressed in terms of implicit function of other variables.
- We can easily find the derivatives of implicit functions, just like the ones for explicit functions.