Integration by parts

Integration by parts is one of frequently used techniques to obtain an integration. From Leibniz rule we have:

$$u\frac{dv}{dx} = \frac{d(uv)}{dx} - \frac{du}{dx}v \tag{1}$$

Integrating both-hand sides by dx, we obtain:

$$\int u\frac{dv}{dx}dx = uv - \int \frac{du}{dx}vdx \tag{2}$$

Integration by parts is very frequently used in physics. When asked which mathematical trick I use most often, I answered "integration by parts."

Problem 1. (Hint[1])

$$\int xe^{2x}\,dx = ? \tag{3}$$

Problem 2. (Hint[2])

$$\int x^2e^{2x}\,dx = ? \tag{4}$$

Problem 3. (Hint[3])

$$\int \ln x\,dx = ? \tag{5}$$

Problem 4. Prove

$$\int e^x \sin x\,dx = -e^x\cos x + \int e^x\cos x\,dx \tag{6}$$

$$\int e^x \cos x\,dx = e^x\sin x - \int e^x\sin x\,dx \tag{7}$$

Thus, show that

$$\int e^x \sin x\,dx = \frac{(\sin x - \cos x)e^x}{2}, \qquad \int e^x \cos x\,dx = \frac{(\sin x + \cos x)e^x}{2} \tag{8}$$

Problem 5. Prove (Hint[4])

$$\int \sin^n x dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x dx \tag{9}$$

Summary

- Integration by parts is given by

$$\int u\frac{dv}{dx}dx = uv - \int \frac{du}{dx}vdx$$

[1] Use $u = x, \quad dv/dx = e^{2x}$

[2] Use integration by parts twice. You will need to use the result of Problem 1.

[3] Use $u = \ln x, \quad dv/dx = 1$

[4] Use $u = \sin^{n-1}x, \quad dv/dx = \sin x$. Then, use $1 - \cos^2 x = \sin^2 x$.